1-Click Brazilian Design Chart for Service and Ultimate Limit States

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Summary

Design chart is associated with reinforced concrete material. The design and verification of members under an eccentric axial load for service and ultimate limit states is a practice guided by several codes like ABNT, CEB-FIB. Usually, they are built from encapsulated data in dimensionless parameters for well-conditioned shapes. The crucial point is how to do the integration over area. Programs performing an analytical integration have been used. Numerical integration is the core of Finite Element Method, and this article presents these techniques to draw design charts, as traditional ones. The data are not encapsulated allowing for any shape, material or bar layout. Literature examples are presented and with 1-Click on screen a popup window display the results. The method's versatility is demonstrated by its extension for any material code (supplying strengths and stress-strain curves). Due to precision, this method solves the design problem with a simple plane region mesh generation.

1 INTRODUCTION

A computer program is always involved when designing structural members subjected to eccentric axial loads. For well-conditioned shapes, [2] presents all the dimensionless equations that form the basis of any computational algorithm used to generate printed design charts, such as those in [4][5][6][8], and [3] explains all limit states. Applications performing quasi-analytical integration, as in [2], or pure analytical integration, as in [11], also solve the problem. Shape difficulties and equilibrium interaction algorithms are the primary limitations identified. Finite element (FE) techniques, derived from the finite element method (FEM) in solid mechanics structural analysis [14][15], solve shape problems and enable modular programming, as demonstrated in programs like these. The simplicity of Gaussian integration formulas helps overcome the challenges of interaction algorithms. The goal is to easily obtain numerous reinforced concrete (RC) design charts for ultimate limit states (ULS) of deformation analysis, including non-homogeneous sections. Additionally, the method can be applied to any of the service limit states (SLS) analysis in a simple manner. Due to the high degree of generalisation and an extensive reference list, the specific properties are highlighted but not repeated. Section 2 introduces the fundamental concepts, with details provided as needed. Sections 3 to 6 are interdependent and form the core of the presentation: THE GROUP, which gathers all data; THE CLOUD, responsible for scanning the entire plane $\mathbb{R}^2$ and storing thousands of results; THE DATA MINING, which extracts hundreds of design charts from the cloud; and THE 1-CLICK, which transforms target input into a popup display. Section 7 is devoted to selected examples, and Section 8 provides the final comments.

2 FUNDAMENTALS

Fig. 1(a) shows the conventions adopted [1][13] for global and local coordinates. A virtual camera is aligned with the local x-axis (following the right-hand rule), focusing on the origin section where only the efforts, nx, my, mz are relevant. These values are always in a vectorial sense—no plane moments or other RC conventions are considered. In (b), a camera view of the cross section with the known terms is shown. The neutral axis (NA) is defined by the plane $\pi = (\vec{n}, d_o)$ in equation (1), where $\vec{p} = (y, z)$ is a vector to a point $p(y, z)$, $\vec{n}$ is the unit normal vector, and d_o is the distance from the origin to the plane. If $\vec{p}$ does not satisfy (1), the remaining values represent the shortest signed distance from the point to the plane.

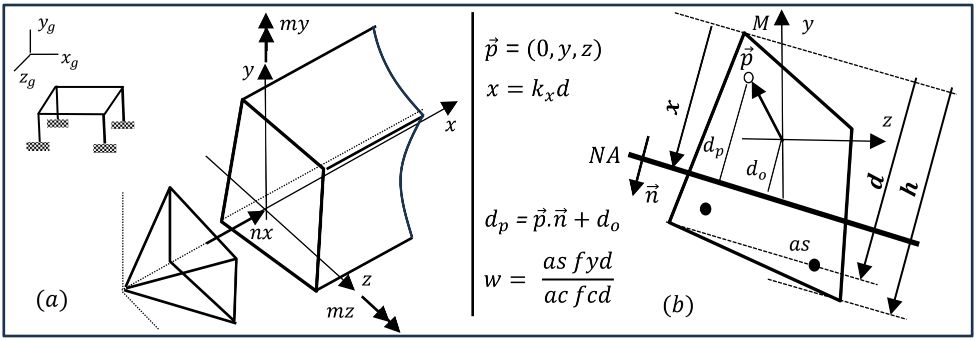

Fig. 1 (a) Global and local coordinates, virtual camera focusing origin; (b) RC cross section: NA and its conventional values: x, k_x the position and dimensionless depth of the NA from top point M, h maximum height, d design height, w mechanical steel rate (as, fyd, ac, fcd are the total areas and design resistances of steel and concrete respectively).

The origin M of x is not suitable for equilibrium purposes; therefore, in Fig. 2, x has its origin at the NA. Together, NA and x form a Cartesian plane used to express the equilibrium.

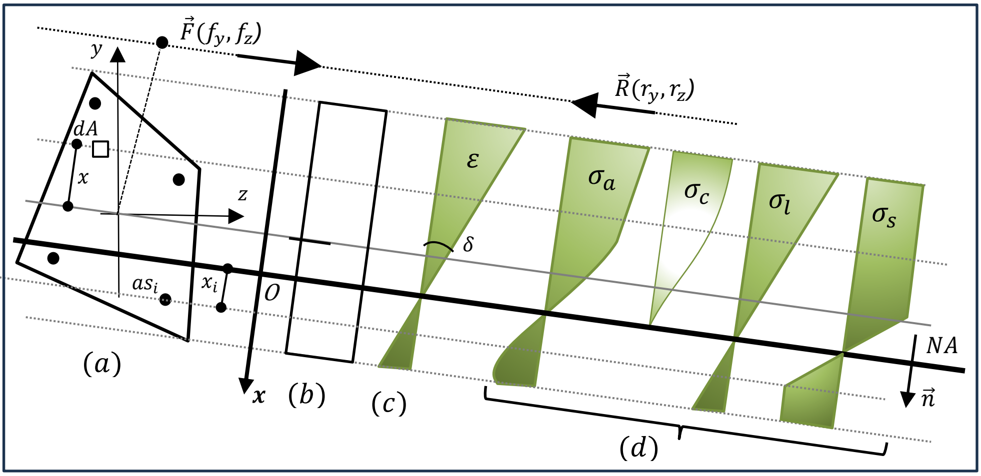

Fig. 2 (a) Cross section under an eccentric axial load $\vec{F}(f_y, f_z)$; (b) the profile; (c) linear diagram of deformation $\varepsilon(x)$; (d) four common material stress diagrams: $\sigma_a(x)$ for a nonlinear material; $\sigma_c(x)$ for compressed concrete (RC); $\sigma_l(x)$ for a linear-elastic material, and $\sigma_s(x)$ for a perfect plastic material.

The linear deformations $\varepsilon(x)$, based on Bernoulli's/Navier's hypothesis (plane sections remain plane under deformation), enable the transformation of coordinates from stress-strain material relations to stress-x coordinates, resembling the material's stress-strain diagrams. Substituting the respective derivatives in the chain rule equation (2) leads to the integral equation (3).

$$\vec{p}.\vec{n} + d_o = 0 \tag{1}$$

$$\frac{d\sigma}{dx} = \frac{d\sigma}{d\varepsilon}\frac{d\varepsilon}{dx} \tag{2}$$

$$\sigma(x) = \int \sigma'(\varepsilon)\ \varepsilon'(x)dx \tag{3}$$

Using the linear stress-strain diagram, that is, $\sigma'(\varepsilon) = E$, the Young's modulus, and $\varepsilon'(x) = k_c$, results in equation (4). This is Hooke's law, as assumed by the Strength of Materials formulas.

$$\sigma(x) = Ek_c x + C, \; k_c x = \varepsilon, \; \boldsymbol{\sigma(x) = E\varepsilon} \quad (C = 0, \quad \sigma'(\varepsilon) \neq 0) \tag{4}$$

To achieve equilibrium, equations (5) and (6) must be satisfied (tol subscript denotes tolerance):

$$|\vec{F}| = |\vec{R}| + R_{tol} = |R_c + R_s| + R_{tol} \tag{5}$$

$$\sqrt{(fy - ry)^2 + (fz - rz)^2} < r_{tol} \tag{6}$$

If the deformation and stress diagrams are known, the resultants of R_c and R_s are:

$$R_c = \int \sigma_c(x) \; dA \tag{7}$$

$$R_s = \sum_{i=1}^{n_s} \sigma_s(x_i) \, as_i \tag{8}$$

All equations are well known in ULS analysis and are valid for any stress-strain curves. If the stresses are in the near-linear deformation zone, as predicted by SLS analysis codes, the same procedures from ULS analysis can be applied. Item 4.3 presents the strategy developed for this proposal.

Both interactive algorithms for solving (5) and (6) and analytical computational integration [2][6][11] are replaced by scanning over the plane of the section to calculate and store each $\vec{R}(r_y, r_z)$. Equation (7) is replaced, in advance and synthesis, by the sum in equation (9). All right-hand side terms are redefined in (4.1). Observe the impressive similarity in equations (8) and (9). This procedure is repeated for every element of the mesh.

$$R_c = \sum_{j,k}^{n} \sigma_c(x_i) \, w_j w_k \det \mathrm{J}(s_j, t_k) \tag{9}$$

3 THE GROUP

The group is an extended concept of a group, gathering all input data and the components generated for it. Four main items form a GROUP: a shape, materials, a layout, and a rhythm. When defined and executed, they constitute the complete description of a cross section, which can be used for pre/post design and/or verification.

3.1 The Shape

A shape has its geometry defined through an FE mesh. These elements are identical to MEF isoparametric elements. In references [14] [15], every detail can be found, including FORTRAN small routines for 4-node bi-linear and 8-node quadratic elements, which are implemented here in C++ as RS4 and RS8. A file with nodal coordinates and element connectivity must be generated. Each element must be linked to a material type.

3.2 The Material

Meshes and layout elements must be attached to a material type. As well as a FE (RS4 or RS8) implementation, materials must also be implemented. Materials have intrinsic and general properties. Intrinsic properties are implemented and can be recognised by a material type code. Generic properties are defined during the input phase. For example, RC material is coded with its stress-strain relation and the domains (corresponding to ultimate deformations, see Section 3.4); resistances and creep coefficients are generic properties specified during the input phase. A FE mesh can work with different materials; for instance, two concretes with different resistances, structural reinforcement of RC beams with a steel plate or other materials, and the combination of a concrete deck and an alloy steel web. Steel bar diameter and resistance can vary, as can their nature. Four material types are implemented: RC concrete, alloy steel, RC steel bars, and PSC strands and wires. The last two are active reinforcement, and in addition to resistances, the initial stresses must also be provided. This is done by defining the levels of initial stress in the SLATE levels (3.4.2)

3.3 The Layout

The layout is almost an FE mesh. It is a mesh of elements with only one node, which is the centre of gravity (y, z) of the steel bars. Each bar is linked to the material type code. One input file defines the layout. The total area of the installed RC steel bars is as_{inst} and the installed rate is w_{inst}.

3.4 The Rhythm

In choosing the rhythm to generate THE CLOUD for the ULS analysis, three rhythm data are essential: rates, angles, and distances. When possible, an arithmetic progression (AP) series defined by a template AP (first, last, step) is used. The steel rates form an AP (≥0, >0, >0), and the angular changes of the NA follow an AP (0°, 360°, >0°). The distances or positions of the neutral axis NA are defined by deformation states (DS), which originate from the domain supplied by the material code. For RC ULS, ABNT defines six domains [3][5]; therefore, it is necessary to determine the number of positions within each domain (e.g., setting the k_x intervals). These positions form the DS for the ULS analysis. If various materials are interacting, the chosen domain must be the innermost one. This occurs in RC domains, which constrain the steel deformations in the compression zone.

3.4.1 The Loads and the Angles

To facilitate the creation of a series of design charts, many values must be anticipated. This enables the extraction of numerous design charts. The colours RED, GREEN, and BLUE correspond to compression, tension, and zero axial loads, respectively. RED loads represent an AP (> 0, > 0, > 0), GREEN loads represent an AP (< 0, < 0, > 0), see Fig. 1(a) shows the actual signals. The BROWN colour corresponds to the angles used to draw design charts with a fixed angular position of the axial load. This is represented as an AP (0°, > 0°, < 360°).

3.4.2 The Pairs and the Levels

The PURPLE colour is associated with SLS analysis. A list of pairs providing the axial service load and the respective steel rate w^*_{req} is expected. Usually, w^*_{req} is calculated using as_{req} from a previous ULS analysis and the resistances of the SLS analysis rather than the design resistances (Fig 1 (b) and item 4.3). Another list of levels defines the slice data (number of slices and data for determining slice spacing) as explained in item 4.3. PURPLE pairs and PURPLE levels have separate input files.

The SLATE colour is associated with prestressed concrete (PSC). Two levels must be supplied to work with PSC materials: the levels of initial prestress and the levels of bond, referring to steel wire and/or strands. Both slate levels also have separate input files (an adapted version has been implemented with one initial prestress and 100% bond).

4 THE CLOUD

Items 4.2 and 4.3 present the construction of the clouds for ULS and SLS analysis, respectively. Before that, the main properties of the FE and its working aspects are briefly outlined. See [14][15] for details and computer implementation, noting that in those references, the natural coordinates in the plane are (r, s), while here they are (s, t).

4.1 The Finite Element

To calculate the integral (7) using the finite element (FE) method, the basic procedure is to represent a point p(y,z) using interpolation functions in the natural coordinate system, i.e., $h_q = h_q(s,t)$ as shown in equation (10), where m is the number of element nodes, and (y_q, z_q) are the coordinates of the nodes. These functions have the fundamental property that its value is 1 in node q and zero at all other nodes. Observe that $\sigma_c(x)$ is a function of x alone. The first step is to determine $\sigma_c(x)$ from a given point $p'(s,t)$ using equation (10), as shown in Fig. 3. The second step is to solve the integral component of equation (7) through numerical integration. The most common integration scheme is Gauss's quadrature, which exactly integrates a polynomial of order $2n-1$, where n is the integration order (IO). There are n^2 pairs of integration points (IP) (s_p, t_p) and weights (w_p) selected a priori to achieve optimal precision. The RC functions σ_c(x) are quadratic, and setting $n = 2$ is sufficient to accommodate any element distortions, IP: $(\pm 1/\sqrt{3}, \pm 1/\sqrt{3})$, w_p: (1.0). Equation (11) presents the analytical integral, its equivalent in natural coordinates (see [14] for $detJ$) and the numerical version, given by two nested sums, as used here. Rewriting equation (11) as equation (12), where each (s_j, t_k) represents the integration points of order n chosen, these can be expanded into equations (13) and (14). For each IP p, σ_p is the tension as given in Fig. 3, da_p is the p-th parcel of dA, and rc_p is equivalent to R_c. The total area of this element is ac. R_c is never calculated, instead, the parcel is passed to a $\vec{R}$ accumulator. Not forgetting equation (8), this process is repeated within a large loop over all the elements of the mesh. This collection generates one surface for each w, and all of them together form THE CLOUD.

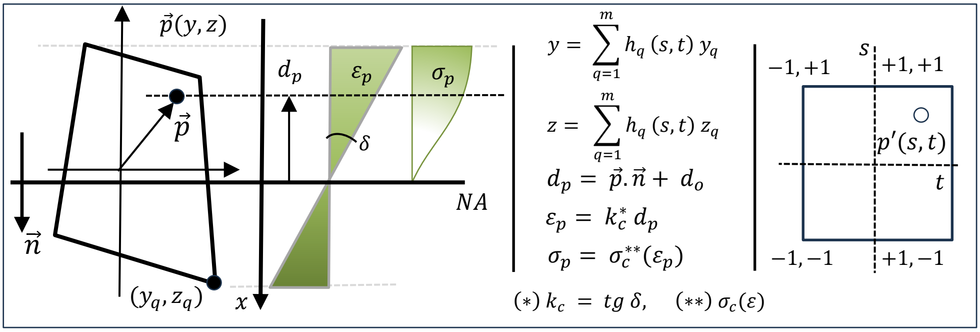

Fig. 3 1-element mesh of the cross section. Determination of tension σ_p at a point $p'(s,t)$.

$$y = \sum_{q=1}^{m} h_q(s,t)\, y_q, \qquad z = \sum_{q=1}^{m} h_q(s,t)\, z_q \tag{10}$$

$$R_c = \int \sigma_c(x)\, dA = \int_{-1}^{+1}\int_{-1}^{+1} \sigma_c(x) detJ\, dr\, ds = \sum_{j=1}^{n}\sum_{k=1}^{n} \sigma_c(x)\, w_j\, w_k detJ(s_j, t_k) \tag{11}$$

$$R_c = \sigma_c(x) \sum_{j=1}^{n}\sum_{k=1}^{n} w_j\, w_k detJ(s_j, t_k) \tag{12}$$

$$rc_p = \sigma_p\, da_p, \quad da_p = w_j w_k detJ(s_j, t_k), \ p = (j-1)\, n + k \tag{13}$$

$$ac = \sum_{p=1}^{n^2} da_p \tag{14}$$

4.2 Ultimate Limits States Cloud

In ULS analysis, reduction factors are applied to material resistance, and the resulting forces are the increased ultimate values. For each steel rate w, the angle of the neutral axis NA ranges from 0° to 360°. For each angle, the NA position ranges from (−∞, +∞), or kx = (−∞, +∞), as explained in section 3.4. At the end of these three nested loops, thousands of key values, such as $w, \vec{n},\ kx, \vec{R},\ r_y, r_z, \varepsilon_c, \varepsilon_s$, etc., are stored. These are all the ultimate values of the section and are referred to as the cloud.

4.3 Service Limits States Cloud

In SLS analysis, instead of reduction factors and increasing coefficients for the loads, the raw material characteristic resistances and service loads are used. The SLS cloud is built from a hypothetical ultimate limit state (HULS), as it starts from a design state (DS) identical to ULS. It is hypothetical because this identity has no relevance to raw material characteristics. For each hypothetical domain, its internal "area" is sliced into a convenient number of slices until the deformation approaches zero. The current level of service is expected to lie between these levels. Refer to Fig. 4(a), where one HULS (the frontier 2-3 [3]) is shown. The dashed lines indicate the "area" sliced into five levels of service, numbered from 0 (the original) to 4. Consider that w_{req}^* is exactly one of the SLS rate series. The side (b) of Fig. 4 shows the first octant of the surface at level 0 for w_{req}^*. For each level, a surface is constructed, the process is completed, and the cloud is ready. These surfaces appear as intersections with the plane

$\pi_v = (\vec{j}, 0)$, ($\vec{j}$ is the axis my unit vector, resembling the dashed lines in Fig. 4(b). If the HULS has n_d DS and n_l is the number of levels then, the total number of the SLS DS is $n_t = n_d . n_l$. In the next step (see the following section item (d)), the design charts are extracted from the intersection of the plane $\pi_h = (\vec{\imath}, ns)$ (where $\vec{\imath}$ is the axis nx unit vector and ns is the service axial load) and are sketched in dashed lines in Fig. 4 (b). The slice intervals can be constant or defined by a geometric progression (GP), determined by the number of terms (slices), the ratio, and the sum of the terms.

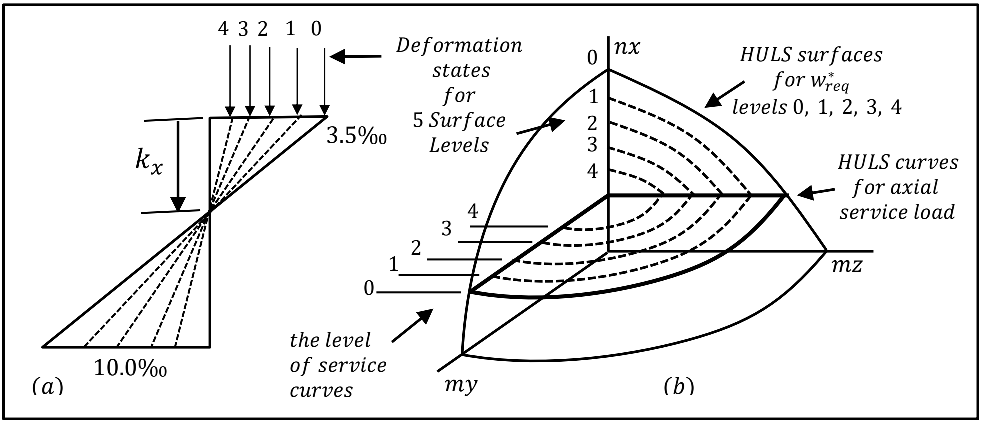

Fig. 4 SLS levels correspond to single HULS DS. (b) Detached first octant of HULS (w^*_{req}).

5 THE DATA MINING

Data mining consists of carefully extraction of data from the clouds. Every design chart is obtained throughout the intersection of the cloud surfaces with a yz-parallel plane π_h or an axis x-parallel plane π_v. The intersections produce isocurves quasi-concentric called kernel of the chart and is where 1-Click happens. The names of previously mentioned colours retain their meaning and are the colours selected to draw the kernels, as explained in items (a)–(e). For each moment chart (m-chart), there is an eccentricity chart (e-chart), except for the BLUE and BROWN ones.

(a) **The Pink Series** consists of charts inspired by old printed charts, where RED and BLUE loads prevailed. Pink charts result from the intersection of π_h planes and the ULS cloud surfaces, producing charts with steel rate w kernels. The π_h position is determined by RED, BLUE, and GREEN loads, as explained in items 3.4.1, with one chart for each load. PINK is the kernel colour.

(b) **The Red and Green Series** are obtained for each steel rate w, that is, by the intersection of π_h with the cloud surface for w, producing charts with the RED and GREEN loads kernel. The π_h-position is determined by RED and GREEN loads.

(c) **The Brown Series** are inspired by [12]. They are an intersection of cloud surfaces with a π_v, the geometric place of the axial loads. The angles are given by BROWN angles (3.4.1). The chart kernels (in BROWN colour) are the steel rates w. They do not follow the conventional camera view fig. 1 (b).

(d) **The Purple Series**. SLS analysis. They are constructed from an extended domain as explain (4.3). The PURPLE pairs (3.4.2) provide the service axial loads and their respective w^*_{req}. For each pair, one chart is drawn, and the kernel (in PURPLE colour) shows all levels of service. If the surface for w^*_{req} has not been constructed, a double interpolation is performed to find the right surface.

(e) **The Slate Series**: This is used with PSC materials. Two SLATE levels control the construction of the SLATE charts: the level of initial stress and the levels of bond, although the latter is not yet functional. However, given a combination of these levels, a pink-blue m-chart can be constructed, where the kernel represents the steel rates w. A common combination is an initial stress and 100% bond.

6 1-CLICK

The 1-Click mechanism is a linear interpolation between the kernel curves. After 1-Click (left mouse button), a solution is found, and a summary dialog window popup. The most important results are the ULS's w_{req} and as_{req} (the rate and as required) and SLS's ε_c and ε_s.

7 EXAMPLES

The original values are expressed inside parenthesis. For example, a value = 20.0 (19.0) means the computed value 20.0 versus the reference value 19.0. The figure captions explain the results, and the abbreviations for the cross-sections (CS) are as follows: R-CS (rectangle), L-CS, I-CS, HR-CS (hollowed rectangle), HP-CS (hollowed parable), T-CS (triangle) C-CS (channel), AT (angular T), π-CS (double T). For ULS analysis of PSC material, ap_{inst} represents the PSC steel area, and if only PSC steel bars are specified, w_{inst} is set to 0.

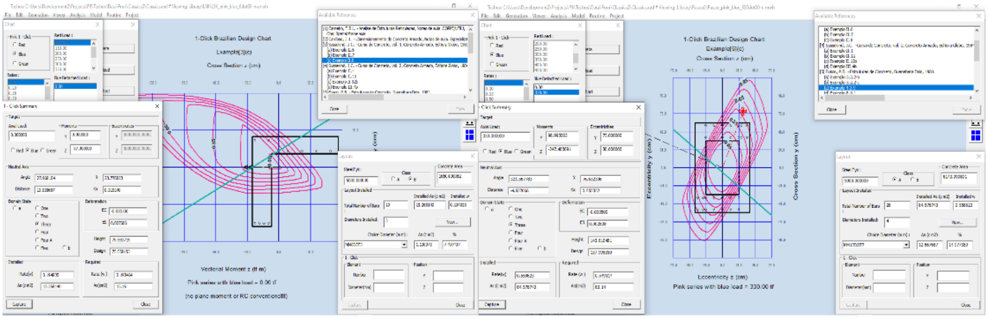

Fig. 5 Left: example II.8 [3] L-CS under a mz_d = -370 kNm (determined by successive attempts). The pink blue m-chart 1-Click view values: w_{req} = 0.19 (0.19). neutral axis angles 27.96° (23.96°). Right: example 4.3.9 [5] an eccentric load on a HR-CS with strong asymmetric layout (solved using the section transformation method); the pink red e-chart shows the values and w_{req} = 0.549 (0.559).

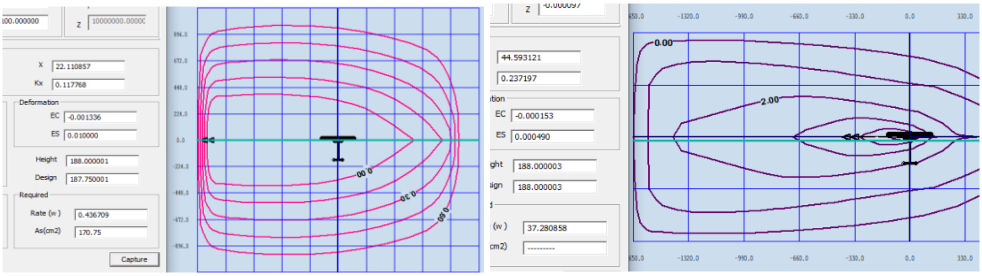

Fig. 6 Example 5.9 [7]. Composite bridge beam with a concrete deck and steel I-shaped web. (a) ULS pink blue m-chart; (b) SLS analysis purple m-chart showing levels 0-6 of service, with the calculated level being 3.73; stresses: top σ_c = 0.57 kN/cm^2 (0.56); bottom σ_s = 10.29 kN/cm^2 (10.33). The values in parentheses are recalculated from original formulas (to keep n = 6, chosen fck = 39 MPa).

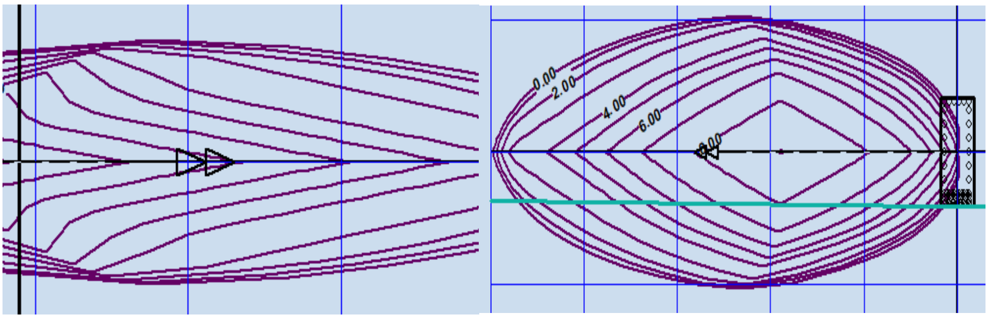

Fig. 7 SLS analysis purple m-chart. Left: R-CS RC example 12 [11]: a purple pair (nx = 0, w^*_{req} = 0.22), ε_c = -0.675‰ and ε_s = 1.21‰. σ_s= 25.41 kN/cm^2 (25.24). Right: R-CS example 6.2 [9]: purple pair (nx = 0, w = 0), ε_c = -0.466‰ and $\varepsilon_s \approx 0$ ‰ (0), NA x-depth = 159.98 cm (160.0) and mz_d = 5433 kNm (5450).

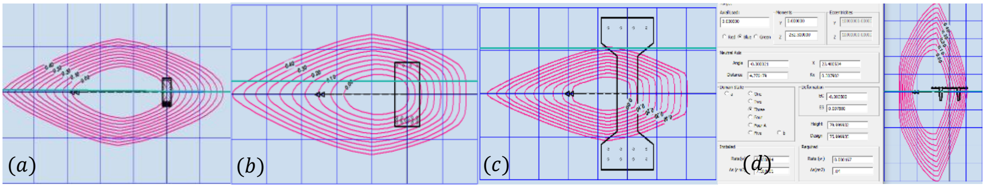

Fig. 8 ULS analysis and PSC. (a) example 6.1[9]: R-CS: w_{req} = 1.27 10^{-4}(0.), ap_{inst} = 48.79 cm^2 (49.15), mz_d = 7632 kNm (7630); (b) example 6.9.1.1[10] R-CS: as_{req} = 25.02 cm^2 (25.25), ap_{inst} = 10.14 cm^2 (10.0), mz_d = 1467.9 kNm (1467.9). ULS and PSC; (c) example 6.9.1.2 [10] I-CS: as_{req} = 4.65 (5.2), ap_{inst} = 4.06 (4), mz_d = 361 kNm (361) (d) example 6.9.1.3[10] π-CS: as_{req} = 5.02 cm^2 (5.2), ap_{inst} = 26.4 cm^2 (26.0), mz_d = 2523 kNm (2505.1).

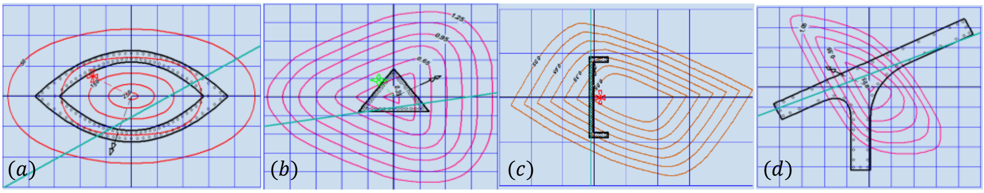

Fig. 9 ULS analysis. (a) HP-CS red e-chart, w = 0.20; (b) T-CS pink green m-chart under an eccentric axial load; (c) C-CS brown m-chart α = 0° (see Figure 12.12 [12]); (d) AT-CS pink blue m-chart.

8 CONCLUSIONS

The results for ULS analysis are excellent and consistently align with those from other methods for RC. For PSC material, the pink-blue m-chart gives values that, when compared with those from examples, are also very accurate. The SLS analysis works very well but is dependent on the correct choice of the number of levels and the spacing between them. RC sections at service have a small NA x-depth, whereas PSC sections typically exhibit the opposite. The complexion of the curves is variable. For a complete PSC material analysis, it is essential to implement various levels of initial prestresses, bond levels, and possibly a prestressed steel rate w_p, similar to the mechanical w of RC material. Although the SLS analysis algorithms can be improved, the values are promising. The widest charts are the BROWN series. Beyond interpolation issues, large cross-sections like shear walls, lock the NA within the section, disrupting all types of algorithms – these were fixed. ULS analysis for RC materials has been extensively tested, and the results are very promising. The actual computational effort is spent on building the clouds, while the others are not substantial, and 1-Click process is instantaneous.

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