IS MONETARY POLICY NEUTRAL IN RELATION TO STOCK RETURNS AND/OR VOLATILITY?

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Abstract: The main objective of this paper is to learn about the monetary policy effects on the B3 return and volatility. Here, monetary policy is proxied by the market interest rate expectation measured by the Brazilian Central Bank. The B3 returns and volatility are daily ones and discounted by the average daily inflation rate. The monetary policy effect on B3 average daily returns is done by testing the Granger causality between them and estimating GARCH models: S-GARCH, E-GARCH, and GJR-GARCH. The results showed that the monetary policy is neutral under Granger causality. Nonetheless, under GARCH estimates, B3 returns as expected are diminished by the monetary policy, but with no effect on its volatility. In other words, expected interest rate changes are well anticipated by the market, which is highly beneficial.

Keywords: Monetary Policy; Volatility; Ibovespa; GARCH.
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1. INTRODUCTION

Monetary policy's main instrument is the setting of interest rates with the aim of controlling inflation in the economy. As a result, it affects the returns on government bonds and variable income assets such as stocks and their indices, such as the B3's IBOVESPA. In order to understand this mechanism of action of monetary policy, authors Elyasiani and Mansur (2005) analyzed the sensitivity of bank stock returns to changes in interest rate levels and volatility using the GARCH-M methodology, concluding that returns are sensitive to both factors. In turn, Guerello (2019) investigated the effect of investor confidence in the transmission mechanism of monetary policy (GARCH), finding a significant and positive relationship between them. Along the same lines, Konrad (2018) evaluated the impact of surprise monetary policy on asset return volatility in Germany (GARCH), finding that it significantly affects volatility. Expanding the analysis, Hsing (2014) examined the effects of fiscal and monetary policies on the stock market in Poland (VAR-GARCH), ultimately understanding that both influence it. Finally, Silva (2019) analyzed how news about monetary policy affects the behavior of

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financial assets in Brazil before and after the global financial crisis using a bivariate VAR-GARCH model, concluding that such news has a significant effect on assets.

Regarding stock returns, Angelico and Oliveira (2007) proposed an ARMA-GARCH model to analyze the temporal precedence between global stock indices, finding that there is an interconnection of financial markets and that the New York and Tokyo stock exchanges influence the Brazilian one. On the other hand, Kobunda and Nogueira Jr. (2017) estimated the volatility of the Ibovespa between 2001 and 2016 through ARCH models, concluding that it is highly persistent and that macroeconomic events can explain the instability of the Brazilian financial market. In turn, Souza-Sobrinho (2006) estimated a GARCH model for Ibovespa data, finding that such a model is efficient for understanding its volatility behavior. Therefore, it is worth noting that all of the cited articles seek to improve the understanding of the dynamics of Ibovespa volatility, providing tools to aid in the development of the present work.

When it comes to volatility, works such as Schwert (1989) and Glosten, Jagannathan and Runkle (1993) demonstrate that such phenomenon is a key factor in asset pricing, being influenced by various variables, such as asymmetric information, market and credit risk, etc. Regarding its estimation, Andersen, Bollerslev, Diebold, and Ebens (2001) highlighted implied volatility as a more precise measure of future volatility, as it incorporates market-available information, such as option prices. On the other hand, historical/statistical volatility is more appropriate as a risk measure, since its estimation is based on past return fluctuations. Seeking a model that aggregates the characteristics of this variable, Engle and Granger (1987) proposed the Autoregressive Conditional Heteroskedasticity (ARCH) model, which became one of the most used approaches in financial literature.

Therefore, in the context of Brazil and based on the cited and studied articles, the present study focused on analyzing the monetary policy (expectations of interest rates) and its impacts on the volatility of the Brazilian stock market's real index (deflated Ibovespa). The S-GARCH, E-GARCH, and GJR-GARCH models were used to understand the statistical relationship of these variables and empirically answer whether monetary policy is neutral, negatively or positively related to returns and volatility of assets traded in the national stock market. A total of 48 models were estimated and classified according to the AIC criterion, enabling the comparison of the efficiency of different GARCH methodologies. Therefore, the next sections are dedicated to the theoretical review, methodology, results, and conclusion of this study.

2. LITERATURE REVIEW

Given the degree of risk involved in variable income operations (such as the stock market), the increase or decrease of the basic interest rate has significant impacts on assets, as agents seek to maximize return and minimize risk. Thus, an increase in this rate encourages investments in fixed income or lower risk assets. Therefore, this section aims to investigate and analyze recent studies on the relationship between monetary policy and financial asset volatility.

2.1 MONETARY POLICY AND ASSETS RETURNS

Several studies have been conducted to examine the relationship between monetary policy and the return of financial assets. In his work, Patelis (2003) used a regression analysis to test the relationship between monetary policy and the predictability of stock returns. Specifically, the author used multiple linear regression models with lagged variables of stock returns, as well as other explanatory variables such as short-term interest rates and the consumer price index. The objective was to evaluate the effect of an unexpected monetary policy on the prediction of stock returns, and the conclusion was that they have a significant impact on the predictability of these returns. In the study conducted by Elyasiani and Mansur (2005), the authors analyzed the sensitivity of the distribution of bank stock returns to variations in long-term interest rates (yield on ten-year government bonds) in terms of both level and volatility (GARCH-M model). The results indicated that the conditional variance of yields has a significant effect on the distribution of these stock returns, and that changes in the level of interest rates also affect the distribution of returns, concluding that the GARCH-M model can be a useful tool to evaluate the sensitivity of bank stocks to changes in interest rates.

In turn, Geromichalos et al. (2017) used a theoretical dynamic general equilibrium model to study the effect of monetary policy on financial asset prices, concluding that expansionary policies tend to raise asset prices and increase their volatility, while contractionary policies tend to reduce their volatility. The study by Laoopdis (2015) adopted a panel data methodology to investigate the dynamics of the stock market in different monetary regimes, finding that monetary policy affects volatility to different degrees depending on the monetary regime in which the country is located. Guerello (2019) investigated the impact of investor confidence on the transmission of monetary policy to short-term interest rates, using a multivariate GARCH model. The results demonstrated that investor confidence has a positive and significant effect on the transmission of monetary policy to short-term interest rates, but had no significant effect on long-term interest rates. Therefore, the result was that investor confidence is an important channel of monetary policy transmission.

The work of Hsing (2014) aimed to analyze the impact of fiscal and monetary policies on the stock market in Poland. To do so, a VAR-GARCH model was used, which is an econometric technique used to estimate the volatility of the stock market. The results indicated that fiscal policy had a positive and significant impact on stock market returns. This means that fiscal measures adopted by the Polish government, such as increasing public spending or reducing taxes, had a positive effect on stock market performance. On the other hand, monetary policy did not have a significant effect on stock market returns. This suggests that measures taken by the Polish Central Bank, such as reducing interest rates or implementing bond-buying programs, did not have a direct impact on stock market performance. However, despite not having a direct effect on stock market returns, monetary policy was identified as one of the influences on stock market volatility. That is, even though monetary policy does not directly affect stock market returns, it can influence the stability and fluctuation of stock prices. Thus, both fiscal and monetary policy may play an important role in the dynamics of the stock market in Poland.

The objective of the study conducted by Konrad (2018) was to analyze the impact of surprises in monetary policy on the volatility of asset returns in Germany. Using a GARCH-M model, an econometric technique to estimate asset price volatility, the study found results that indicate that surprises in monetary policy significantly increase the volatility of asset returns in Germany. Additionally, the volatility of asset returns is also influenced by the volatility of long-term interest rates and macroeconomic uncertainty. Factors such as political uncertainty, instability in international markets, and changes in investor expectations can also affect the volatility of asset prices. The study concludes that monetary policy has a significant impact on the volatility of asset returns in Germany. This suggests that measures taken by the German Central Bank can directly affect the stability and fluctuation of asset prices, with important implications for the financial market and the economy as a whole.

Finally, Silva (2019) conducted research that examined the impact of news about monetary policy on the behavior of financial assets in Brazil, before and after the global crisis of 2008, using a bivariate VAR-GARCH model for analysis. The results indicated that news about monetary policy has significant effects on the volatility of financial assets, regardless of the period analyzed. This implies that information disclosed by the Central Bank of Brazil, such as changes in interest rates, for example, can cause significant changes in financial asset prices. Additionally, the research found that the financial market's reactions to news about monetary policy changed after the 2008 crisis. This suggests that the financial market has become more sensitive to such news, possibly due to the greater uncertainty and volatility generated by the crisis.

Therefore, it is observed that the results and conclusions for different countries are the same (statistically significant) when it comes to the relationship between monetary policy and the volatility of financial assets (which can affect stock market returns - the topic of the next section of this work).

2.2 BRAZILIAN STOCK MARKET RETURNS

The stock market is a market that attracts the attention of many investors, either because of the possibility of high returns or because of the ease of access through brokerage firms. However, it is important to bear in mind that as a variable income asset, it is possible to obtain negative positions that, when executed, result in losses to the investment portfolio. To better understand the behavior of the index of shares of the largest national companies, Angelico and Oliveira (2007) conducted a study using the ARMA-GARCH model, verifying the influence of the New York and Tokyo stock markets on the Brazilian financial market. To do so, the authors examined the relationship between the stock indices of these markets at different periods of time, seeking to identify a possible temporal precedence between them. The results of the study indicated an interconnection of the financial markets, with the New York and Tokyo stock exchanges significantly influencing the Brazilian market. This means that the fluctuations and events that occur in foreign financial markets directly affect the performance of the Brazilian market. Going further, Silva (2012) used an ARIMA-GARCH approach to analyze which variables affect the return of the IBOVESPA and its volatility, and his results indicated that its conditional variance is influenced by both external factors, such as international volatility, and domestic macroeconomic events, such as changes in monetary policy and political uncertainties.

Using conditional heteroskedasticity models (GARCH) and stochastic volatility (Local Scale Model - LSM), Santos (2012) calculated the Value at Risk (VaR) for the IBOVESPA post-2008 crisis. The author concluded that VaR can be a useful tool for managing risks in stock market investments (it helps investors and risk managers determine the maximum potential loss that could be suffered in a given period of time). In this way, it is possible to establish acceptable loss limits and adopt strategies to manage and mitigate risks. However, it is important to remember that these statistical models do not consider the possibility of extreme events, that is, rare and unpredictable events that can have a significant impact on the stock market. Therefore, although VaR is a useful tool, it should not be used in isolation and should be combined with other risk management techniques to ensure adequate protection against significant losses.

Finally, another study by Kobunda and Nogueira Jr. (2017) analyzed the returns of the IBOVESPA between 2001 and 2016 using ARCH models. The results showed that the volatility of the IBOVESPA is highly persistent and that the instability of the Brazilian financial market can be explained by macroeconomic events, such as the 2008 financial crisis. In summary, investing in the stock market can bring attractive returns, but it is important to be aware of market volatility. The cited studies show the importance of analyzing the risks involved in such assets to make predictions about future returns.

2.3 VOLATILITY: CONCEPTS AND REFERENCES

Volatility, or conditional variance, is an important measure for the asset market as it reflects the degree of uncertainty about expected returns and is used to assess the risk associated with a financial asset. It reflects the fluctuation of prices over time, so the higher the level of volatility, the riskier the asset associated with it. The work of Glosten, Jagannathan, and Runkle (1993) proposed a model to analyze the relationship between the expected value and the volatility of excess nominal stock returns. The model they proposed, called GARCH-M, considers that the current volatility of stock returns depends not only on the current expected value but also on the past expected value and past conditional variance. Conditional variance is the variance of the excess residuals (i.e., the returns after subtracting the expected return) over a certain period of time. The GARCH-M model suggests that future volatility is more influenced by past volatility than by past expected value. The authors concluded that past volatility has a significant impact on future volatility of excess nominal stock returns. This means that if an asset's historical volatility is high, it is more likely that future volatility will also be high, regardless of the expected value of excess returns.

In turn, Schwert (1989) analyzed the relationship between stock volatility (in the US market) and various economic and financial factors, including economic activity, financial leverage (multiplication of income through debt), and stock trading activity (using monthly data from 1857 to 1987). He concludes that financial leverage has a significant correlation with stock volatility, but other factors, including political uncertainty and changes in economic activity, can also influence fluctuations in stock volatility. Additionally, the author highlights that the high variability of stock returns during the Great Depression is difficult to explain with simple stock valuation models, suggesting that other factors may also play an important role in stock market volatility.

The behavior of conditional variance is linked to the distribution of asset returns, so Andersen, Bollerslev, Diebold, and Ebens (2001) proposed a model to describe the relationship between realized volatility and stock returns using high-frequency data. Realized volatility is a measure of the volatility that actually occurred in the past, based on historical asset prices, while implied volatility is a theoretical measure of expected future volatility. The authors showed that the distribution of stock returns is characterized by a heavy tail, meaning that extreme events are more common than would be expected in a normal distribution (most of the data is concentrated around the mean, and there is less data in the extremes). This indicates, according to the model, that extreme market movements may be more frequent than expected and that investor uncertainty may be greater than anticipated.

2.4 CONTRIBUTIONS

The empirical results allow for the evaluation of whether monetary policy is neutral, negatively, or positively related to the returns and volatility of assets traded on the national stock market. In this context, the present research aimed to analyze the relationship between monetary policy and the volatility of the Brazilian stock market, focusing on expectations of changes in interest rates. To this end, GARCH models (widely recognized in the literature as useful tools for modeling and forecasting volatility in financial series) were used. A total of 48 models were estimated, following the S-GARCH (Equation 2), E-GARCH (Equation 3), and GJR-GARCH (Equation 4) methodologies using the daily return of the Ibovespa and the interest rate expectation measures provided by the Central Bank of Brazil as an exogenous variable (with a total of 2822 daily samples between January 1, 2012, and January 30, 2023):

$$R_t = \sigma_t e_t \quad \text{(Conditional mean)} \tag{1}$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i R_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma^2{}_{t-j} \quad \text{(S-GARCH)} \tag{2}$$

$$ln(\sigma|t^2) = \alpha_0 + \sum_{i=1}^{q} \frac{|R_{t-i}| + \delta_i R_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{p} \beta_j ln(\sigma^2{}_{t-j}) \quad \text{(E-GARCH)} \tag{3}$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} (\alpha_i + \gamma_i N_{t-i}) R_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma^2{}_{t-j} \quad \text{(GJR-GARCH)} \tag{4}$$

Equation (1) represents the conditional mean of the series R_t (Ibovespa returns), which is equal to the product of the conditional volatility σ_t with an error term e_t. Equation (2) represents the conditional variance of the series σ_t^2, which is a function of the constant α_0, past squared returns R_t and past squared conditional volatility $\sigma^2{}_{t-j}$. Equation (3) is a variation of equation (2), in which the term $\frac{|R_{t-i}| + \delta_i R_{t-i}}{\sigma_{t-i}}$ is added to incorporate asymmetry in the conditional volatility. Finally, equation (4) is an extension of equation (2), in which the term N_{t-i} is a dummy that takes a value of one if $R_{t-i} < 0$ and a value of zero otherwise (see Section 3 - Methodology).

The lags of the models are defined by the value of q (conditional mean) and p (conditional variance), indicating that the higher this value, the longer the past effects on volatility. The estimated models use the Ibovespa index volatility as the dependent variable and daily interest rate expectations (mean and variance) as exogenous variables. They can be divided into four groups where the first one (Table 1) does not include exogenous variables and uses only the index's volatility to estimate its conditional variance.

Table 1 - Estimated models (S-GARCH, E-GARCH, and GJR-GARCH) without exogenous variable and their respective parameters.

Models	Methods	Estimated parameters
1	sGARCH	mu, ar1, ma1, alpha1, beta1, shape, omega
2	eGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, omega
3	gjrGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, omega
4	sGARCH	mu, ar1, ma1, alpha1, beta1, shape, omega, ma2
5	eGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ma2, omega
6	gjrGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ma2, omega
7	sGARCH	mu, ar1, ma1, alpha1, beta1, shape, omega, ar2
8	eGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ar2, omega
9	gjrGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ar2, omega
10	sGARCH	mu, ar1, ma1, alpha1, beta1, shape, ma2, ar2, omega
11	eGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ma2, ar2, omega
12	gjrGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ma2, ar2, omega

Note. Mu = α_0
Source: Own autorship.

The second group (Table 2) considers both the conditional mean and variance as dependent on the exogenous variable, that is, the daily interest rate. In this case, the index volatility is modeled together with the interest rate to estimate the mean and conditional variance of the time series.

Table 2 - Estimated models (S-GARCH, E-GARCH, and GJR-GARCH) with the mean and variance of interest rate expectations as exogenous variables and their respective parameters.

Models	Methods	Estimated parameters
1	sGARCH	mu, ar1, ma1, alpha1, beta1, shape, omega, mxreg1, vxreg1
2	eGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, omega, mxreg1, vxreg1
3	gjrGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, omega, mxreg1, vxreg1
4	sGARCH	mu, ar1, ma1, alpha1, beta1, shape, omega, ma2, mxreg1, vxreg1

5	eGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ma2, omega, mxreg1, vxreg1
6	gjrGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ma2, omega, mxreg1, vxreg1
7	sGARCH	mu, ar1, ma1, alpha1, beta1, shape, omega, ar2, mxreg1, vxreg1
8	eGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ar2, omega, mxreg1, vxreg1
9	gjrGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ar2, omega, mxreg1, vxreg1
10	sGARCH	mu, ar1, ma1, alpha1, beta1, shape, ma2, ar2, omega, mxreg1, vxreg1
11	eGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ma2, ar2, omega, mxreg1, vxreg1
12	gjrGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ma2, ar2, omega, mxreg1, vxreg1

Note. Mu = α_0
Source: Own autorship.

The third (Table 3), in turn, considers only the conditional variance as dependent on the exogenous variable. That is, the daily interest rate is used to estimate the variation of the Ibovespa index volatility over time.

Table 3 - Estimated models (S-GARCH, E-GARCH, and GJR-GARCH) with only the variance of interest rate expectations as an exogenous variable and their respective parameters.

Models	Methods	Estimated parameters
1	sGARCH	mu, ar1, ma1, alpha1, beta1, shape, omega, vxreg1
2	eGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, omega, vxreg1
3	gjrGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, omega, vxreg1
4	sGARCH	mu, ar1, ma1, alpha1, beta1, shape, omega, ma2, vxreg1
5	eGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ma2, omega, vxreg1
6	gjrGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ma2, omega, vxreg1
7	sGARCH	mu, ar1, ma1, alpha1, beta1, shape, omega, ar2, vxreg1
8	eGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ar2, omega, vxreg1
9	gjrGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ar2, omega, vxreg1
10	sGARCH	mu, ar1, ma1, alpha1, beta1, shape, ma2, ar2, omega, vxreg1

11	eGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ma2, ar2, omega, vxreg1
12	gjrGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ma2, ar2, omega, vxreg1

Note. Mu = α_0
Source: Own autorship.

The fourth group (Table 4) considers only the conditional mean as dependent on the exogenous variable. This means that the daily interest rate is used to estimate the expected value of the Ibovespa index volatility, while the variation of the time series is modeled autoregressively.

Table 4 - Estimated models (S-GARCH, E-GARCH, and GJR-GARCH) with only the mean of interest rate expectations as an exogenous variable and their respective parameters

Models	Methods	Estimated parameters
1	sGARCH	mu, ar1, ma1, alpha1, beta1, shape, omega, mxreg1
2	eGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, omega, mxreg1
3	gjrGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, omega, mxreg1
4	sGARCH	mu, ar1, ma1, alpha1, beta1, shape, omega, ma2, mxreg1
5	eGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ma2, omega, mxreg1
6	gjrGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ma2, omega, mxreg1
7	sGARCH	mu, ar1, ma1, alpha1, beta1, shape, omega, ar2, mxreg1
8	eGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ar2, omega, mxreg1
9	gjrGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ar2, omega, mxreg1
10	sGARCH	mu, ar1, ma1, alpha1, beta1, shape, ma2, ar2, omega, mxreg1
11	eGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ma2, ar2, omega, mxreg1
12	gjrGARCH	mu, ar1, ma1, alpha1, beta1, shape, gamma1, ma2, ar2, omega, mxreg1

Note. Mu = α_0
Source: Own autorship.

The classification of the models was carried out using the AIC criterion (widely used in the literature to compare the efficiency of different statistical models), and its results can be observed both in Section 4 - Results and in APPENDIX “A”, “B”, “C” and “D” - Complete Results. Therefore, the following sections of the article are dedicated to deepening the methodology employed in the empirical analysis, presenting the results, and drawing conclusions.

3. METHODOLOGY

3.1 DATA COLLECTION

The IBOVESPA is the main performance indicator of stocks traded on B3, the index was created in 1968 and functions as a signal of the performance of the national capital market (B3, 2023). Being a guide for Brazilian and foreign investors, the IBOVESPA is influenced by a series of factors ranging from political to economic events. Thus, the study used daily closing data taken from the Yahoo!Finance website for the period from January 3, 2013 to January 31, 2023, to calculate the return. The models were estimated based on the real variable, that is, the inflation of the day was discounted. Therefore, the return on day t of the IBOVESPA index was calculated as follows:

$$Rt = ln(Pt) - ln(Pt\text{-}1)^{4} \quad (6)$$

The use of returns, instead of prices, for model estimation is due to their statistical properties. According to Tsay (2010), many financial studies estimate models with this variable because it is a series that, in many cases, is stationary, unlike prices. The condition of stationarity is an important determinant because it ensures that the mean, variance, and autocovariance do not vary over time, leading to the mean reversion process, which is nothing more than the variable returning to the series' mean. Therefore, a stationary series fluctuates around its mean with a constant amplitude (Cuthbertson, Hall & Taylor, 1995).

The Selic rate expectations were taken from the Central Bank's "Market Expectations System," which has statistics calculated daily based on the market expectations of approximately 130 financial agents (such as banks, brokers, managers, etc.). The survey, which began in 1999, monitors expectations to facilitate decisions regarding monetary policy. Based on this information, an indicator variable for Selic expectations was constructed, and it was used as an exogenous variable in the estimated models. The first step was to calculate the daily mean, median, and standard deviation of these expectations to obtain a single value that captured the market sentiment. It should be noted that the basic rate is fixed annually, as are its expectations, but considering that IBOVESPA returns are daily, the daily Selic rate expectation was sought, that is, the daily value of the rate. To this end, the difference between the previously obtained means was calculated, resulting in:

$$ExpSelic = MS_t - MS_{t-1} \quad (7)$$

In which, *MS* represents the average of the Selic rate expectations on day t and t-1, and ExpSelic is the exogenous variable indicating the value of daily Selic rate expectations.

3.2 GRANGER CAUSALITY TEST

In order to understand the relationship between monetary policy and IBOVESPA return during the study period, a Granger Causality Test was performed. This test checks whether one variable, such as ExpSelic, causes another variable, such as R. The test therefore analyzes how past values of ExpSelic explain current values of R, and can be

[4] Ln = Natural logarithm, which is the logarithm with base e.

carried out using a Vector Autoregressive (VAR) model according Mantalos e Shukur (2010). Once the VAR is constructed, causality can be tested by linear estimation of each equation in the model, based on the method of Ordinary Least Squares (OLS). The null hypothesis of no causal relationship between the variables is then established.

There is an important point to consider when conducting the Granger test. Since the traditional model uses the OLS method to check the relationship between variables, the presence of GARCH effects in the series affects the test result. Some studies - such as Mansson and Shukur (2009), Mantalos et al. (2007) and Mansson (2012) - have found that in these cases the true null hypothesis tends to be over-rejected. The degree of the GARCH model should also be considered, as series with GARCH effects greater than (1,1) present more problems. The presence of GARCH effect is common in financial series, so caution should be exercised when analyzing the results of the Granger test and be aware of the rejection of H0.

3.3 ARCH AND GARCH MODELS

One of the great characteristics of financial series is the presence of volatility, defined as the variance of conditional variation over time according Filho et. al (1993). Given the importance of this factor for investors and market agents as a whole, econometric models from the ARCH and GARCH family are increasingly gaining space in the literature.

Revolutionizing volatility studies, Engle (1982) proposed the Autoregressive Conditional Heteroskedasticity (ARCH) model, which assumes that the variance of a financial time series changes over time, conditioned on errors observed in the past. In other words, it seeks to estimate time-dependent volatility as a function of past observed volatility according Perlin et al. (2021). Volatility is therefore given by conditional variance and can be described by an ARCH(m) process defined by the following equations:

$$R_t = \sigma_t e_t \quad \text{(Conditional mean)} \tag{8}$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 R_{t-1}^2 + \cdots + \alpha_q R_{t-m}^2 \text{ (Conditional variance)} \tag{9}$$

The return is represented by the variable Rt, while the conditional volatility at time t is given by σ_t^2, and α is the estimated parameter that represents the reaction time of the conditional variance. In the conditional mean equation, e_t represents a sequence of identically distributed random variables. The lag of the model is defined by the value of q, indicating that the longer the past effects on volatility will be. The parameters are commonly estimated using maximum likelihood method and nowadays, with the aid of statistical software, this process is almost instantaneous.

In order to simplify the ARCH model, since its estimation in practice depends on a significant number of lags which increases the number of estimated parameters and makes it difficult, Bollerslev (1986) proposed the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model. The model developed by Bollersley allows the conditional variance to be an ARMA process, which practically makes an ARCH(q) equivalent to a GARCH(p,q). Thus, the GARCH model defines volatility as a function of the squared past errors (ARCH component) plus past conditional variances.

The specification of a GARCH(p,q) process is therefore determined by the equations according Perlin et al. (2021):

$R_t = \sigma_t e_t$ (Conditional mean) (10)

$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i R_{t-i}^2 + \sum_{j=1}^{p} \beta_j {\sigma^2}_{t-j}$ (Conditional variance) (11)

As in the ARCH(m) model, e_t remains a white noise term with mean 0 and variance 1. The term α_i represents the ARCH parameter in the model, denoting q as the degree of the ARCH process, while the degree of the GARCH process is denoted by p, and β_j indicates how much volatility felt in the previous period persists in the current moment. It should be noted that the following conditions must be established for a consistent estimation:

1. $\alpha_0 > 0$
2. α_i e $\beta_j \geq 0$
3. $\alpha_i + \beta_j < 1$

The restriction on positive parameters ensures that the variance is finite, and the stationarity condition is given by $\alpha_i + \beta_j < 1$. Although the described conditions are necessary for the estimation of a parsimonious model, they are not always observed since financial series are marked by a complex dynamic with asymmetric effects (leverage effect), volatility clusters, and fat tails according Marques (2017). The leverage effect, or volatility asymmetry, refers to the behavior of volatility in response to different events. This implies that it reacts differently depending on the event, meaning that good and bad news have distinct impacts on it. Thus, periods marked by price drops (with bad news) are marked by high volatility, while good news tends to decrease volatility according Perlin et al. (2021).

Volatility clusters, or groups of volatility, describe a common phenomenon in financial series. Periods of intense volatility are followed by equally intense periods, meaning that there is persistence in shocks. Thus, events that increase volatility in period t-1 tend to affect volatility in period t. Finally, another characteristic of these series is the presence of fat tails, referring to the fact that, in many cases, the distribution of returns is not normal. When this is verified, it is understood that the probability of extreme events occurring is significant. Seeking to accommodate and study these characteristics, the GARCH and ARCH models advanced, giving rise to a series of other models within the same family such as the EGARCH and GJR-GARCH models. In 1991, with the aim of capturing the characteristics discussed above, Daniel Nelson proposed the Exponential Generalized Autoregressive Conditional Heteroskedasticity (EGARCH) model, enabling the verification of asymmetric effects on volatility according Nelson (1991).

Given that the EGARCH model is built on a logarithmic basis, there are no restrictions on the parameters, since the logarithmic function itself is restricted to positive values. Therefore, it is possible to verify the behavior of volatility in response to different shocks without the need for parametric restrictions according Marques (2017). The EGARCH models the conditional variance as follows:

$$ln(\sigma|t^2) = \alpha_0 + \sum_{i=1}^{q} \frac{|R_{t-i}|+\delta_i R_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{p} \beta_j ln(\sigma^2{}_{t-j}) \text{ (Conditional variance)} \quad (12)$$

The conditional variance becomes an asymmetric function of the past values of Rt according Marques (2017). The asymmetry of Rt is measured by the parameter δ_i, implying that negative returns increase volatility more than positive returns. Another model that is able to capture the asymmetric effects in the conditional variance is the GJR-GARCH, developed by Glosten, Jagannatha and Runkle (1993), which is structured as follows:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} (\alpha_i + \gamma_i N_{t-i}) R_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma^2{}_{t-j} \text{ (Conditional variance)} \quad (13)$$

In it, N_{t-i} is a dummy that takes a value of one if $R_{t-i} < 0$ and zero otherwise according Perlin et al. (2021). From this, a negative return impacts volatility by $(\alpha_i + \gamma_i)$ R_{t-i}^2 and a positive return by $\alpha_i R_{t-i}^2$. The leverage effect is therefore captured by the parameter γ_i. It can be concluded that both models, although different, are capable of capturing effects on volatility not considered in the traditional GARCH model. These are just a few of the models that make up the GARCH family, there is a diversity of models that will not be addressed as this study only estimated the ARCH, SGARCH, EGARCH, and GJR-GARCH models. After estimating the models, it was necessary to find their best specification, that is, the one that best describes the data, and for that, there are several criteria, one of which is the Akaike Information Criterion (AIC). Proposed by Akaike in 1974, the AIC is a function in which the quality of the fitted model is penalized depending on the number of estimated parameters. From this, the model chosen for the results analysis was the one that presented the lowest AIC value according Perlin et al. (2021).

4. ANALYSIS OF RESULTS

This section aims to discuss the results obtained from the estimation of the models and their tests. To do so, a series of analyses about the behavior of IBOVESPA returns is necessary. First, the Jarque-Bera test was performed to check the distribution of the series, whose null hypothesis is that the data follows a normal distribution. For the variable analyzed, the p-value found (2.2e-16) indicates the need for rejection of this hypothesis when the p-value associated with the test is less than a predetermined level of significance (usually 0.05). The results obtained from both the Jarque-Bera test and the skewness and kurtosis indicate that the t-student distribution more accurately describes the data. Table 5 presents the main information about the return series.

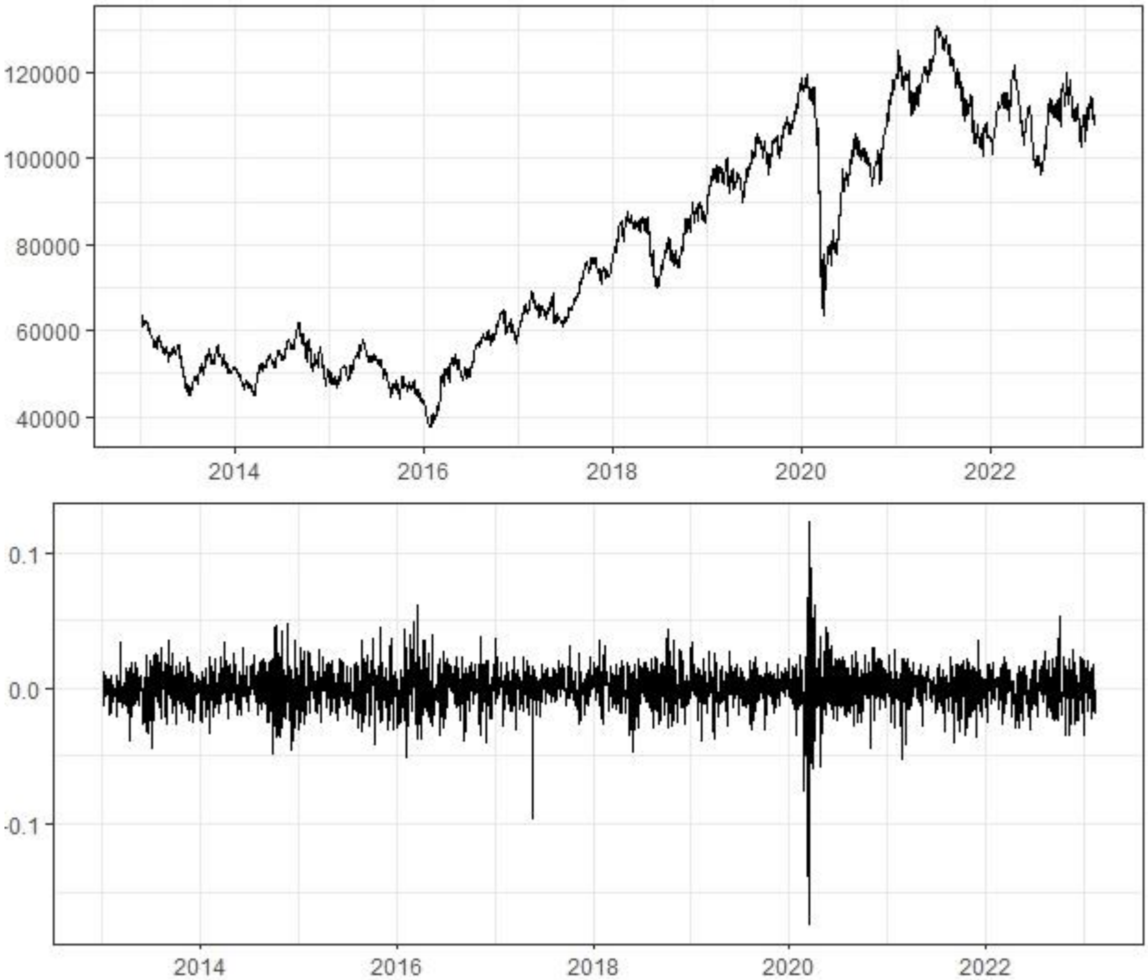
Table 5 - Descriptive statistics

Variable	Max	Mín	Skewness	Kurtosis	Jaque-Bera (p-value)
Retorno.IBOV	0,1221002	-0,173429	-1,245189	18,22565	2.2e-16

Source: Own elaboration with data from Yahoo!Finance website.

As can be seen in Figure 01-A, the index price has a trajectory marked by successive highs and lows, with the largest one in 2020. This phenomenon is reflected in the IBOVESPA return, as shown in Figure 01-B, indicating the presence of heteroscedasticity in the series. The ARCH-LM test according Engle (1982) provides formal proof of heteroscedasticity in returns for up to 4 lags.

Figure 1 - BVSP Behavior



Source: Own elaboration with data from Yahoo!Finance website.

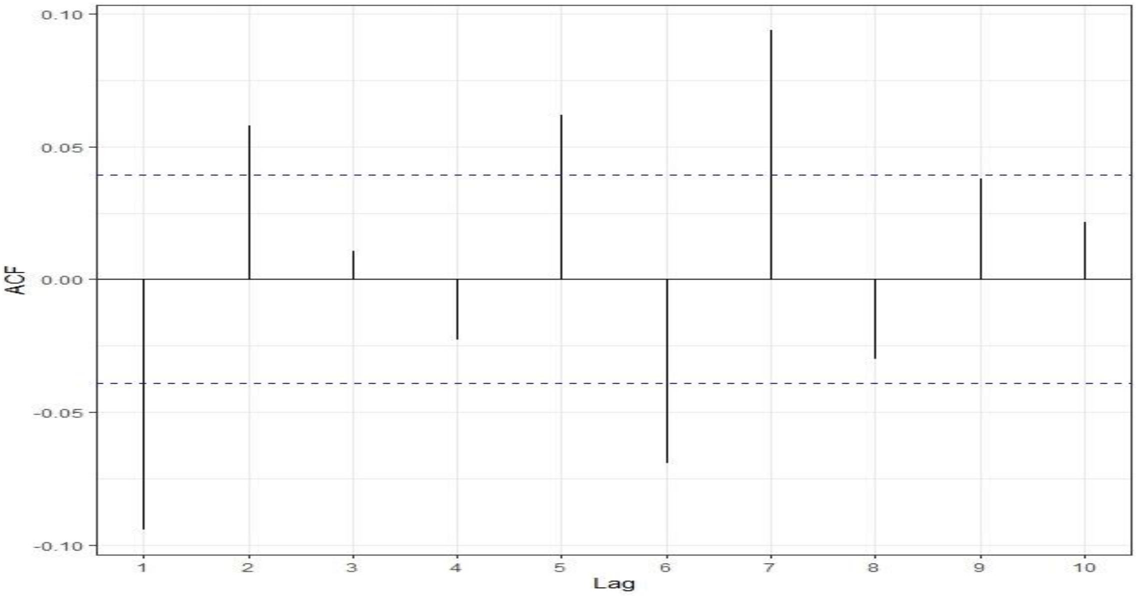
Engle's (1982) test assumes the null hypothesis of the absence of ARCH effects in the series, i.e., the absence of conditional heteroscedasticity. Considering that GARCH models are indicated for heteroscedastic series, the confirmation of this phenomenon is of utmost importance in the pre-estimation phase. Table 6 shows the test results for up to 3 lags, and since the p-value found was less than 2.2e-16, the null hypothesis is rejected, confirming the existence of heteroscedasticity.

Table 6 - ARCH-LM test results

Lag	ARCH-LM	p-value (less than)
1	532.8	2.2e-16
2	917.55	2.2e-16
3	929.95	2.2e-16

Source: Own elaboration with data from Yahoo!Finance website.

Figure 2 - Autocorrelation function (Ret BVSP)



Source: Own elaboration with data from Yahoo!Finance website.

In addition to heteroscedasticity, the series presents other characteristics to be considered for model estimation. Thus, Figure 2 presents the autocorrelation function for returns, confirming its existence. Autocorrelation occurs when an observation at time *t* is correlated with an observation at time *t-k*, indicating that past occurrences have implications in the present according Perlin et al. (2021). Based on the presented information, it is possible to proceed with the Granger test and the estimation of GARCH models, considering the t-student distribution as the standard and market expectations of the Selic rate as the exogenous variable.

4.1 GRANGER CAUSALITY

As previously stated, verifying the existence of causality between variables is an important point, as it helps to understand the behavior and relationship between them. However, caution is needed when performing the Granger test since the ARCH effect is present in the series under study. Table 7 shows the result of the Granger causality test between real returns (Ibovespa) and monetary policy.

Table 7 - Granger Test

Models	Df	Pr(>F)
Ret. BVSP ~ Exp Selic	-2	0.1658
Exp Selic ~ Ret. BVSP	-2	0.5554

Source: Own elaboration.

Where (*Df*) refers to the order used or the number of lags; and Pr(>F) demonstrates the p-value found. The test has as a null hypothesis the absence of a causal relationship between the variables, and considering that the p-value is greater than the significance level of 0.05, there is not enough evidence to reject it. The results of the Granger causality test, presented in Table 03, indicate that there is not enough evidence to claim that monetary policy in the period "caused" the daily returns of Ibovespa, and the reverse is also true. The first model tests the hypothesis that monetary policy causes the daily return of Ibovespa, reaching a p-value of 0.1658, higher than the significance level (0.05). The second analyzes the inverse hypothesis, that is, that the return causes monetary policy, and in this case, the p-value found (0.5554) was even higher. Therefore, the applied causality test indicates neutrality between the variables.

4.2 EFFECTS OF MONETARY POLICY ON VOLATILITY

The estimation of the models was divided into 4, resulting in 48 estimated models presented as an annex in this work. The first group considered only the real return of Ibovespa, while in the second, third, and fourth, the Expselic exogenous variable was added. Table 8 shows the best models of each group using AIC value as the selection criterion.

Table 8 - Model Results

	Model 1	**Model 2**	**Model 3**	**Model 4**
mu	0.001	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)
omega	0.000***	-0.221***	0.000***	-0.238***
	(0.000)	(0.007)	(0.000)	(0.009)
alpha1	0.069***	-0.066***	0.030***	-0.068***
	(0.004)	(0.012)	(0.007)	(0.013)
beta1	0.898***	0.974***	0.896***	0.972***
	(0.009)	(0.001)	(0.008)	(0.001)
gamma1		0.119***	0.072***	0.124***
		(0.016)	(0.019)	(0.013)
vxreg1		-0.142	0.000	
		(0.128)	(0.000)	
mxreg1		-0.011*		-0.011*
		(0.005)		(0.005)
Model GARCH	sGARCH	eGARCH	gjrGARCH	eGARCH
Distribution	std	std	std	std
Log likelihood	6955,707	6959,767	6962,985	6966,274
AIC	-5.582	-5.584	-5.586	-5.589

Note: ***p < 0.001; **p < 0.01; *p < 0.05.
Source: Own elaboration.

In the presented models, the values of mu represent the average return of the period given by the conditional mean equation. In all cases, it is not significant and statistically equal to zero, indicating that, in the considered period, the actual return of IBOVESPA was close to zero. The parameter omega represents the part of volatility that is independent of other variables, presenting statistical significance in all four cases. On the other hand, alpha1, in the traditional model and in the GJR-GARCH, measures the effect of a variation in R_{t-i} on daily volatility, while beta1 measures the persistence of volatility. Finally, the asymmetry in conditional variance is captured by gamma1.

Model 1 was estimated using the traditional GARCH (2,2) methodology, which is referred to as model 10 in Annex A. In this model, the three parameters - omega, alpha1, and beta1 - were statistically significant and positive, respecting the restriction imposed by the methodology, and the stationarity condition given by *(alpha1+beta1) < 1* was met. The beta1 parameter, whose value was 0.898, indicates that the main component of explanation of the present volatility is past volatility. Although this model presented good results, it is not capable of capturing the asymmetric effects present in conditional variance, while the other three models estimated from the EGARCH and GJR-GARCH methodologies are.

In the last three models in Table 08, all parameters were significant, implying the presence of asymmetry and volatility clustering in the returns of IBOVESPA. The first indicates that negative returns increase conditional variance, that is, a negative shock increases the index's risk. In EGARCH, it is represented by alpha1 and gamma1, where the former indicates the direction of the shock, and the latter its size. In the GJR-GARCH model, its value is given by the sum of the alpha1 and gamma1 parameters, being 0.102 in this case. Although the determination of asymmetry is different in the two methods, its estimated value was very close in all three cases. The high value of beta1 captures the effect of volatility clustering, indicating that volatility generates volatility. These effects are characteristic of the Brazilian financial system, which is marked by high uncertainty and shock propagation according Perlin et al. (2021).

Analyzing the effects of the exogenous variable, it is concluded that, contrary to the result presented in the Granger test, ExpSelic has statistical significance in the equation of the conditional mean of returns. The parameter that captures this effect is mxreg1, and it is significant at 5%, implying that in the period in question, monetary policy negatively affected the average real return. That is, it was not neutral with respect to the average returns of IBOVESPA. Thus, the neutrality relationship found when performing the traditional Granger test was not confirmed. The different results found between the Granger test and the GARCH models make it clear that the presence of the latter in the series affects the performance of the former.

The negative relationship between monetary policy and IBOVESPA returns is explained by the behavior of agents operating in the market. This is because individuals seek to maximize their returns and minimize risks, so the expectation of an increase in interest rates causes many to migrate their investments, previously allocated in the capital market, to fixed-income assets. This movement directly affects the price of assets and, consequently, their returns. The parameter vxreg1, considered in the estimates of models 2 and 3, represents the impact of monetary policy on volatility, and in both cases, it did not present statistical significance.

This result was already expected considering that the proxy used here for monetary policy derives from the average of agents' expectations. The effect of this on assets depends on the surprise factor, which means that asset volatility is unlikely to react to anticipated policies. Studies such as Durham (2007), Konrad (2009), Bernanke and Kuttner (2005), among others, indicate that the effect of monetary policy anticipated by agents on conditional variance is statistically insignificant. However, there is a change when considering the "surprise" component, where individuals are not expecting variations in monetary policy, it begins to affect volatility.

5. CONCLUSION

This article aims to empirically analyze the relationship between monetary policy and the Brazilian stock market, seeking to identify possible impacts that monetary policy may have on stock market volatility. To do so, the methodology consisted of using measures of expectations of interest rates from the Central Bank of Brazil as an exogenous variable with a total of 2822 daily samples from January 2012 to January 2023 for the IBOVESPA. Two variables were constructed and studied: the average daily return of the IBOVESPA and market expectations about Selic as a proxy for monetary policy. Then, the Granger causality test and the construction of 48 S-GARCH, EGARCH, and GJR-GARCH models were performed.

Through the analysis of the results, it was found that monetary policy negatively affects the real average return of the IBOVESPA but is neutral regarding volatility. These results are relevant for investors who wish to make informed decisions in the Brazilian stock market as they provide an empirical analysis of the relationship between monetary policy and market volatility. Additionally, the results obtained contribute to the advancement of knowledge in the financial area as the methodology used and the results obtained can be applied to future studies related to the Brazilian capital market.

Furthermore, it is important to highlight that the divergences found in traditional Granger and GARCH models indicate that the application of the former is not suitable for time series where the GARCH effect is present. Moreover, the presence of asymmetry and clustering in the volatility of the Brazilian market shows that negative shocks have a greater impact on conditional variance. The clustering factor reflects the persistence of the Brazilian capital market, indicating that past volatility is an important component for explaining current volatility. Therefore, this study is an important contribution to the financial literature as the results can help guide policies and investment strategies in the Brazilian stock market. By presenting an empirical analysis of the relationship between monetary policy and the stock market, the obtained results can assist investors in making informed decisions and contribute to the understanding of the dynamics of the Brazilian capital market.

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APPENDIX A - GARCH MODEL RESULTS WITHOUT EXOGENOUS VARIABLE.

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9	Model 10	Model 11	Model 12
mu	0.001*	0.000	0.000	0.001*	0.000	0.000	0.001*	0.000	0.000	0.001	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ar1	0.360	0.207***	0.320	0.273	0.204***	0.248	0.272	0.876***	0.241	0.911***	-0.113**	-0.088
	(0.356)	(0.032)	(0.440)	(0.587)	(0.032)	(0.650)	(0.548)	(0.034)	(0.590)	(0.006)	(0.044)	-2.346
ma1	-0.392	-0.237***	-0.350	-0.304	-0.235***	-0.277	-0.302	-0.904***	-0.269	-0.927***	0.083	0.058
	(0.350)	(0.033)	(0.435)	(0.587)	(0.033)	(0.650)	(0.547)	(0.034)	(0.590)	(0.007)	(0.044)	-2.346
omega	0.000***	-0.233***	0.000***	0.000***	-0.233***	0.000***	0.000***	-0.229***	0.000***	0.000***	-0.233***	0.000***
	(0.000)	(0.011)	(0.000)	(0.000)	(0.011)	(0.000)	(0.000)	(0.010)	(0.000)	(0.000)	(0.011)	(0.000)
alpha1	0.067***	-0.062***	0.030***	0.067***	-0.062***	0.030***	0.067***	-0.069***	0.030***	0.069***	-0.062***	0.030***
	(0.004)	(0.012)	(0.007)	(0.004)	(0.012)	(0.007)	(0.004)	(0.014)	(0.007)	(0.004)	(0.012)	(0.007)
beta1	0.901***	0.972***	0.900***	0.901***	0.972***	0.900***	0.901***	0.973***	0.900***	0.898***	0.972***	0.900***
	(0.009)	(0.001)	(0.008)	(0.009)	(0.001)	(0.008)	(0.009)	(0.001)	(0.008)	(0.009)	(0.001)	(0.008)
shape	7.141***	7.062***	7.205***	7.134***	7.054***	7.195***	7.134***	7.080***	7.200***	6.954***	7.054***	7.197***
	(0.868)	(0.813)	(0.885)	(0.867)	(0.810)	(0.883)	(0.867)	(0.818)	(0.884)	(0.824)	(0.811)	(0.884)
gamma1		0.125***	0.068***		0.125***	0.067***		0.123***	0.068***		0.125***	0.068***
		(0.023)	(0.018)		(0.023)	(0.018)		(0.022)	(0.018)		(0.023)	(0.018)
ma2				-0.006	-0.000	-0.004				0.983***	-0.094*	-0.141
				(0.029)	(0.020)	(0.029)				(0.000)	(0.046)	(0.761)
ar2							-0.007	0.038***	-0.005	-0.977***	0.086	0.129
							(0.028)	(0.011)	(0.027)	(0.005)	(0.046)	(0.685)
Variance Model	sGARCH	eGARCH	gjrGARCH	sGARCH	eGARCH	gjrGARCH	sGARCH	eGARCH	gjrGARCH	sGARCH	eGARCH	gjrGARCH
Distribution	std	std	std	std	std	std	std	std	std	std	std	std
Log likelihood	6.948.702	6.956.666	6.955.839	6.948.609	6.956.480	6.955.690	6.948.572	6.957.196	6.955.652	6.955.707	6.956.448	6.955.645
AIC	-5.578	-5.583	-5.583	-5.577	-5.583	-5.582	-5.577	-5.583	-5.582	-5.582	-5.582	-5.581
BIC	-5.562	-5.565	-5.564	-5.558	-5.562	-5.561	-5.558	-5.562	-5.561	-5.561	-5.558	-5.558

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

APPENDIX B - GARCH MODEL RESULTS WITH EXOGENOUS VARIABLE (MEAN AND VARIANCE).

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9	Model 10	Model 11	Model 12
mu	0.001*	0.000	0.000	0.001*	0.000	0.000	0.001*	0.000	0.000	0.001*	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ar1	0.371	0.187**	0.329	0.307	0.216***	0.281	0.299	0.866***	0.271	-0.498	-0.094**	0.193
	(0.349)	(0.060)	(0.503)	(0.581)	(0.054)	-1.084	(0.548)	(0.196)	(0.780)	(0.471)	(0.036)	-1.252
ma1	-0.404	-0.220***	-0.361	-0.340	-0.250***	-0.312	-0.331	-0.897***	-0.301	0.466	0.061	-0.224
	(0.343)	(0.059)	(0.496)	(0.581)	(0.054)	-1.084	(0.547)	(0.174)	(0.780)	(0.466)	(0.037)	-1.247
mxreg1	-0.009	-0.011*	-0.010*	-0.009	-0.011	-0.010*	-0.009	-0.010*	-0.010*	-0.009	-0.011*	-0.010*
	(0.005)	(0.005)	(0.005)	(0.005)	(0.006)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)
omega	0.000***	-0.221***	0.000***	0.000***	-0.222***	0.000***	0.000***	-0.218***	0.000***	0.000***	-0.222***	0.000***
	(0.000)	(0.007)	(0.000)	(0.000)	(0.006)	(0.000)	(0.000)	(0.006)	(0.000)	(0.000)	(0.007)	(0.000)
alpha1	0.065***	-0.066***	0.028***	0.065***	-0.066***	0.028***	0.065***	-0.072***	0.028***	0.065***	-0.066***	0.028***
	(0.004)	(0.012)	(0.007)	(0.004)	(0.012)	(0.007)	(0.004)	(0.014)	(0.007)	(0.004)	(0.012)	(0.007)
beta1	0.903***	0.974***	0.902***	0.903***	0.974***	0.902***	0.903***	0.974***	0.901***	0.903***	0.974***	0.901***
	(0.008)	(0.001)	(0.008)	(0.008)	(0.001)	(0.008)	(0.008)	(0.001)	(0.008)	(0.008)	(0.001)	(0.008)
vxreg1	0.000	-0.142	0.000	0.000	-0.142	0.000	0.000	-0.142	0.000	0.000	-0.142	0.000
	(0.000)	(0.128)	(0.000)	(0.000)	(0.128)	(0.000)	(0.000)	(0.126)	(0.000)	(0.000)	(0.128)	(0.000)
shape	7.114***	7.110***	7.202***	7.107***	7.102***	7.193***	7.099***	7.134***	7.193***	7.120***	7.104***	7.188***
	(0.868)	(0.848)	(0.886)	(0.867)	(0.847)	(0.885)	(0.864)	(0.856)	(0.885)	(0.870)	(0.847)	(0.884)
gamma1		0.119***	0.069***		0.119***	0.069***		0.117***	0.069***		0.119***	0.069***
		(0.016)	(0.018)		(0.015)	(0.018)		(0.015)	(0.018)		(0.017)	(0.018)
ma2				-0.005	0.001	-0.003				-0.279	-0.089	-0.035
				(0.030)	(0.022)	(0.043)				(0.364)	(0.083)	(0.597)
ar2							-0.005	0.038***	-0.004	0.246	0.081	0.030
							(0.029)	(0.010)	(0.033)	(0.370)	(0.082)	(0.565)
Variance Model	sGARCH	eGARCH	gjrGARCH	sGARCH	eGARCH	gjrGARCH	sGARCH	eGARCH	gjrGARCH	sGARCH	eGARCH	gjrGARCH
Distribution	std	std	std	std	std	std	std	std	std	std	std	std
Log likelihood	6.950.470	6.959.767	6.957.933	6.950.369	6.959.571	6.957.777	6.950.327	6.960.034	6.957.737	6.950.431	6.959.539	6.957.733
AIC	-5.578	-5.584	-5.583	-5.577	-5.583	-5.582	-5.577	-5.584	-5.582	-5.576	-5.583	-5.581
BIC	-5.557	-5.561	-5.560	-5.553	-5.558	-5.556	-5.553	-5.558	-5.556	-5.550	-5.555	-5.553

***p < 0.001; **p < 0.01; *p < 0.05

APPENDIX C - GARCH MODEL RESULTS WITH EXOGENOUS VARIABLE (VARIANCE).

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9	Model 10	Model 11	Model 12
mu	0.001*	0.000	0.000	0.001*	0.000	0.000	0.001*	0.000	0.000	0.001*	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ar1	0.360	0.196***	0.320	0.277	0.199***	0.247	0.274	0.875***	0.876***	-0.504	0.249***	0.911***
	(0.356)	(0.029)	(0.440)	(0.582)	(0.030)	(0.648)	(0.547)	(0.036)	(0.089)	(0.469)	(0.027)	(0.006)
ma1	-0.392	-0.227***	-0.349	-0.308	-0.230***	-0.276	-0.304	-0.903***	-0.903***	0.473	-0.279***	-0.927***
	(0.350)	(0.029)	(0.435)	(0.582)	(0.030)	(0.648)	(0.546)	(0.036)	(0.085)	(0.465)	(0.028)	(0.006)
omega	0.000***	-0.225***	0.000***	0.000***	-0.226***	0.000***	0.000***	-0.221***	0.000***	0.000***	-0.226***	0.000***
	(0.000)	(0.008)	(0.000)	(0.000)	(0.008)	(0.000)	(0.000)	(0.007)	(0.000)	(0.000)	(0.008)	(0.000)
alpha1	0.067***	-0.065***	0.030***	0.066***	-0.065***	0.030***	0.066***	-0.072***	0.028***	0.066***	-0.065***	0.030***
	(0.004)	(0.012)	(0.007)	(0.004)	(0.012)	(0.007)	(0.004)	(0.014)	(0.007)	(0.004)	(0.012)	(0.007)
beta1	0.901***	0.973***	0.900***	0.901***	0.973***	0.900***	0.901***	0.974***	0.900***	0.901***	0.973***	0.896***
	(0.009)	(0.001)	(0.008)	(0.009)	(0.001)	(0.008)	(0.009)	(0.001)	(0.008)	(0.009)	(0.001)	(0.008)
vxreg1	0.000	-0.150	0.000	0.000	-0.150	0.000	0.000	-0.151	0.000	0.000	-0.150	0.000
	(0.000)	(0.131)	(0.000)	(0.000)	(0.131)	(0.000)	(0.000)	(0.129)	(0.000)	(0.000)	(0.131)	(0.000)
shape	7.141***	7.114***	7.203***	7.122***	7.107***	7.198***	7.122***	7.135***	7.198***	7.142***	7.108***	7.012***
	(0.868)	(0.841)	(0.885)	(0.869)	(0.839)	(0.885)	(0.869)	(0.856)	(0.885)	(0.874)	(0.839)	(0.838)
gamma1		0.121***	0.068***		0.121***	0.067***		0.119***	0.072***		0.121***	0.072***
		(0.018)	(0.018)		(0.018)	(0.018)		(0.016)	(0.019)		(0.018)	(0.019)
ma2				-0.006	0.000	-0.004				-0.276	0.031	0.983***
				(0.029)	(0.020)	(0.029)				(0.361)	(0.056)	(0.000)
ar2							-0.006	0.038**	0.032	0.244	-0.030	-0.977***
							(0.028)	(0.012)	(0.020)	(0.366)	(0.058)	(0.005)
Variance Model	sGARCH	eGARCH	gjrGARCH	sGARCH	eGARCH	gjrGARCH	sGARCH	eGARCH	gjrGARCH	sGARCH	eGARCH	gjrGARCH
Distribution	std	std	std	std	std	std	std	std	std	std	std	std
Log likelihood	6.948.702	6.957.300	6.955.839	6.948.772	6.957.106	6.955.690	6.948.735	6.957.854	6.955.714	6.948.816	6.957.060	6.962.985
AIC	-5.577	-5.583	-5.582	-5.576	-5.582	-5.581	-5.576	-5.583	-5.581	-5.576	-5.581	-5.586
BIC	-5.558	-5.562	-5.561	-5.555	-5.559	-5.558	-5.555	-5.559	-5.558	-5.552	-5.556	-5.560

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

APPENDIX D - GARCH MODEL RESULTS WITH EXOGENOUS VARIABLE (MEAN).

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9	Model 10	Model 11	Model 12
mu	0.001*	0.000	0.000	0.001*	0.000	0.000	0.001*	0.000	0.000	0.001	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ar1	0.370	0.195***	0.330	0.303	0.226***	0.281	0.300	0.867***	0.865***	1.581***	0.914***	-0.519
	(0.351)	(0.035)	(0.502)	(0.585)	(0.058)	-1.082	(0.549)	(0.124)	(0.114)	(0.015)	(0.008)	(0.698)
ma1	-0.403	-0.229***	-0.361	-0.336	-0.260***	-0.312	-0.332	-0.898***	-0.894***	-1.613***	-0.929***	0.489
	(0.345)	(0.036)	(0.495)	(0.585)	(0.059)	-1.082	(0.548)	(0.119)	(0.111)	(0.000)	(0.009)	(0.694)
mxreg1	-0.009	-0.011*	-0.010*	-0.009	-0.011*	-0.010*	-0.009	-0.010*	-0.010*	-0.009	-0.011*	-0.010*
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)
omega	0.000***	-0.226***	0.000***	0.000***	-0.226***	0.000***	0.000***	-0.223***	0.000***	0.000***	-0.238***	0.000***
	(0.000)	(0.009)	(0.000)	(0.000)	(0.009)	(0.000)	(0.000)	(0.008)	(0.000)	(0.000)	(0.009)	(0.000)
alpha1	0.066***	-0.064***	0.028***	0.066***	-0.063***	0.028***	0.066***	-0.069***	0.027***	0.066***	-0.068***	0.028***
	(0.004)	(0.012)	(0.007)	(0.004)	(0.012)	(0.007)	(0.004)	(0.014)	(0.007)	(0.004)	(0.013)	(0.007)
beta1	0.903***	0.973***	0.902***	0.903***	0.973***	0.902***	0.903***	0.974***	0.901***	0.902***	0.972***	0.902***
	(0.008)	(0.001)	(0.008)	(0.009)	(0.001)	(0.008)	(0.009)	(0.001)	(0.008)	(0.009)	(0.001)	(0.008)
shape	7.126***	7.046***	7.201***	7.115***	7.039***	7.194***	7.104***	7.070***	7.208***	7.059***	6.842***	7.201***
	(0.867)	(0.822)	(0.885)	(0.865)	(0.820)	(0.884)	(0.862)	(0.835)	(0.887)	(0.853)	(0.858)	(0.887)
gamma1		0.121***	0.069***		0.121***	0.069***		0.120***	0.072***		0.124***	0.069***
		(0.019)	(0.018)		(0.020)	(0.018)		(0.018)	(0.019)		(0.013)	(0.018)
ma2				-0.005	0.001	-0.003				0.713***	0.986***	-0.261
				(0.030)	(0.019)	(0.043)				(0.000)	(0.000)	(0.374)
ar2							-0.005	0.038	0.031	-0.679***	-0.980***	0.233
							(0.029)	(0.020)	(0.020)	(0.015)	(0.007)	(0.380)
Variance Model	sGARCH	eGARCH	gjrGARCH	sGARCH	eGARCH	gjrGARCH	sGARCH	eGARCH	gjrGARCH	sGARCH	eGARCH	gjrGARCH
Distribution	std	std	std	std	std	std	std	std	std	std	std	std
Log likelihood	6.950.364	6.959.160	6.957.933	6.950.263	6.958.970	6.957.777	6.950.222	6.959.419	6.957.621	6.951.310	6.966.274	6.957.774
AIC	-5.578	-5.585	-5.584	-5.578	-5.584	-5.583	-5.578	-5.584	-5.583	-5.578	-5.589	-5.582
BIC	-5.560	-5.564	-5.563	-5.557	-5.560	-5.559	-5.556	-5.561	-5.559	-5.554	-5.563	-5.556

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$