Financial Networks and Systemic Risk: optimal regulation decisions

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Abstract

The objective of this term-paper is to examine the significance of systemic risk in the aftermath of the 2008 financial crisis and more recent events such as the SVB Financial Group crisis. The paper aims to determine the optimal regulatory choices of ex-ante regulation, ex-post bailouts, or laissez-faire approaches. To accomplish this, I utilized the methodology introduced by Jackson and Pernoud (2019) by employing simulated data to compute the optimal regulatory decisions. Apart from that I also compare various Financial Centrality Measures while assessing contagion effects through stress simulations. The results highlight the critical role of networks in financial system analysis and the need for effective policy-making in regulating financial systems to prevent catastrophic scenarios, underscoring the importance of prudent regulatory measures.

1 Introduction

Systemic risk is highly relevant in today's world, as it refers to the potential for a failure or disruption in one part of the financial system to have significant and far-reaching consequences for the wider economy. The interconnectedness of financial institutions and markets, as well as the increasing complexity and speed of financial transactions, make it more challenging to identify and mitigate systemic risk. In recent years, several events have highlighted the importance of systemic risk management, such as the global financial crisis of 2008, the COVID-19 pandemic, and the recent crises surrounding the Silicon Valley Bank (SVB) Financial Group. Therefore, it is crucial for policymakers and financial institutions to monitor and address systemic risk to maintain financial stability and avoid potential economic downturns.

The Basel III framework has significantly enhanced the quality and quantity of capital held by banks, improved risk management practices, and addressed systemic risk in the aftermath of the 2008 financial crisis. To understand and analyze systemic risk more effectively, network analysis has emerged as a powerful tool. It enables modeling and analysis of the complex interactions and interdependencies between financial institutions and markets, thereby facilitating a more comprehensive understanding of systemic risk.

In network analysis, financial institutions and markets are represented as nodes in a network, with links or edges representing the various connections and interactions between them. Network can help to analyze systemic risk through the identification of key players or nodes that are highly connected or central to the network. These nodes, known as "systemically important institutions," are more likely to have a large impact on the rest of the network if they were to fail or experience financial difficulties. This subject has been discussed in numerous paper in the recent years (Sun et al., 2021; Battiston et al., 2021 and 2012a; Barucca et al., 2020; Hałaj and Kok, 2019; Leduc and Thurner, 2018; Cont et al., 2013).

In a financial network, banks and other financial institutions are highly interconnected through a variety of channels. As a result, the externalities created by the failure or distress of one bank can have significant spillover effects on other banks and financial institutions in the network, potentially leading to a broader systemic crisis. Many studies have demonstrated that by examining the structure and dynamics of the network, it is possible to gain insights into how shocks and contagion effects can propagate through the financial system, potentially leading to systemic risk (Liu and He, 2020; Acemoglu et al., 2015; Allen and Gale, 2000; Gai and Kapadia, 2010a).

Overall, network analysis is a valuable tool for understanding and analyzing systemic risk, and it has the potential to improve our ability to identify and mitigate potential sources of risk in the financial system. By identifying and monitoring critical nodes, regulators can focus on maintaining stability through prudential regulations, stress testing, emergency liquidity facilities, and resolution frameworks. However, the presence of moral hazard must be considered as financial institutions may take on excessive risk due to the existence of capital buffers or the expectation of government bailouts.

In this paper, I started from a methodology proposed by Jackson and Pernoud (2019 and 2021) to investigate the optimal regulation decisions, namely, whether to regulate ex-ante, employ ex-post bailouts, or adopt a laissez-faire approach. Additionally, the paper evaluates and contrasts various Financial Centrality Measures and concludes with simulations of multiple stress scenarios and their potential contagion effects.

The results in this work underscore the critical role of networks in the analysis of financial systems and emphasize the crucial need for effective policy-making in regulation. In the absence of sound policies, the emergence of catastrophic scenarios becomes a looming possibility, making it all the more imperative to prioritize the implementation of regulatory measures.

The paper is structured into four main sections. The introduction sets the stage for the study, while the second section covers the methodology and data. This section, apart from a presentation of the data set creation procedures, includes the model for analyzing optimal regulation decisions, financial networks, financial centrality measures, and stress test methodology. Moving on to section three, I present the results of the analysis. Finally, section four concludes the study and outlines future steps.

2 Methodology and data

This section aims to outline the methodology and data set employed in this paper. Initially, I present a general intuition regarding the model to analyze optimal regulation decisions. Following that, the Financial Networks will be introduced, highlighting its unique features and defining it. Next, important definitions and equations for this study will be briefly presented. The financial centrality measures, both traditional and related to systemic risk theory, will then be defined. After that, I go deeper in my application to the proposed Jackson and Pernoud (2019) model for optimal regulation decisions. In the sequence, the stress tests and contagion effects methodology will be discussed. Finally, I present the procedures I used to create the data set to be used in this paper.

I used mostly the software R version 4.2.2 with R-Studio 2022.12.0.

2.1 The intuition of the model

To effectively address systemic risk, it is necessary to take a comprehensive view of the network. To highlight the importance of examining the details of the network in order to identify the key institutions that require regulation or a bailout, I will provide an example from Jackson and Pernoud's (2021) survey research.

To facilitate understanding, the authors consider a network in which banks are connected solely through debt contracts. A measure of systemic risk based on local balance sheet information would only take into account the face value of each bank's assets and liabilities, without considering the identities of their counterparties. However, this approach is inadequate, as demonstrated by the authors' example of a financial network where two banks have identical balance sheets, but their defaults have vastly different consequences.

Therefore, if the central authority had the ability to bail out only one of the two institutions, it would not be able to make an optimal decision based on local information alone.
Consider the network shown in **Figure 1**. Suppose that the portfolios of both Bank 1 and Bank 4 have yielded zero returns, rendering them insolvent. Let Bank 2 earn a return on its portfolio that falls between 3D/4 and D, while Bank 3 earns a return below D/4, and Bank 5 earns a return above D/2. Assuming a recovery rate of zero on assets of a defaulting bank, despite having the same balance sheet, only Bank 1 would cause a widespread default contagion if it remains insolvent. Banks 2 and 3 have sufficient buffers to absorb the shock of Bank 4's default, but not that of Bank 1. Therefore, **bailing out Bank 1 would prevent the entire system from insolvency**, while **bailing out Bank 4 would not have any impact**, and a full systemic failure would occur.

Furthermore, this example highlights the fact that without network information, it is impossible to identify which banks are at risk of insolvency. For example, if one were to examine the books of Bank 3 without knowing that Bank 2 is exposed to Bank 1, then even if one were aware of the portfolio realizations of Bank 3's counterparties, it would not be apparent that Bank 3 is in danger of insolvency. While this is a straightforward example, it demonstrates why regulatory agencies that lack visibility into certain parts of the network (such as foreign institutions or shadow banks) or only have access to data from local stress tests are at a disadvantage.

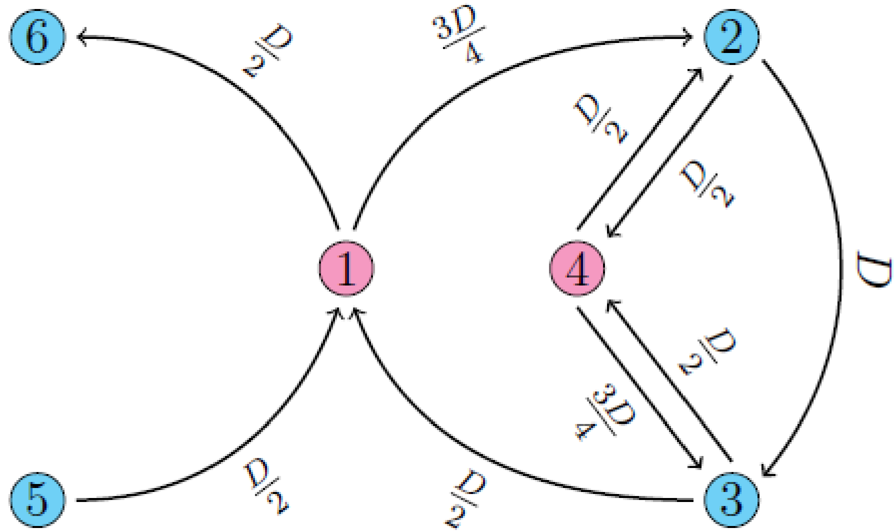


Figure 1: Example on the importance of network information (from Jackson and Pernoud, 2021)

To address this issue, I will use a model the authors proposed in another paper (Jackson and Pernoud, 2021).

The authors suggest considering a scenario where a regulator needs to decide on the most appropriate way to regulate bank i's investments. This decision should take into account the financial network $g = (D, S)$ and the equilibrium investments of other banks q_{-i}, which may or may not be regulated.

Assuming the regulator has the power to intervene in two ways to minimize the inefficiency of banks' investments, there are two options available: first, the regulator can implement a macroprudential policy that restricts the types of investments banks can make. Alternatively, the regulator can opt for ex post bailouts of insolvent banks at some cost. These two interventions result in different investment decisions, and thus lead to varying returns on banks' investments as well as default risks. The authors explore this trade-off between ex post bailouts and portfolio regulation policies, comparing them to a laissez-faire scenario where banks are left to make their own investment decisions and are not bailed out in case of default.

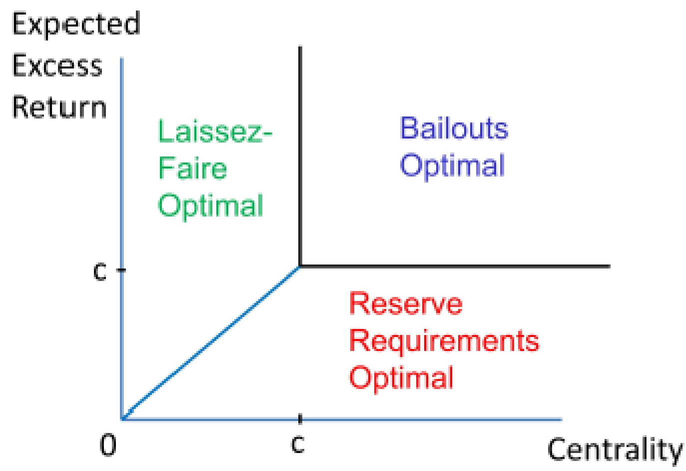
In this template, the macroprudential policy can be implemented in different ways, such as a reserve requirement that mandates a minimum amount of risk-free assets to be held, an upper bound on the variance of a bank's portfolio, or a limit on the amount of the portfolio that is at risk. However, for the purpose of the analysis, the macroprudential policy is specifically defined as an upper bound $\bar{q}_i$ on the share of bank i's portfolio that can be invested in the risky asset.

The presence of discontinuities in bankruptcy costs at certain values of q_i means that even small changes in i's investment can lead to a different set of defaulting banks in some states of the world (I will discuss it in more details later). As a result, there are multiple optimal levels of investment that must be compared to determine the best course of action.

To maximize social welfare, a regulator should impose limits on a bank's portfolio when its risk premium falls below a certain threshold or when its net financial centrality exceeds a certain threshold. Therefore, an optimal macroprudential policy should be discretionary and take into account the potential returns that a bank could generate as well as its position in the network.

Finally, suppose the regulator can also choose to bail out bank i in case of insolvency. However, this action comes at an expected cost of $c_i > 0$ which includes not only the net capital injection but also any additional indirect costs that are not recoverable.

The effects of restricting a portfolio or providing bailouts are equivalent in that they prevent a bank from defaulting. Therefore, the expected bankruptcy costs associated with either option are affected by the extent to which they prevent bank i from defaulting and the resulting effects on the overall financial system. **Figure 2** provides a summary of these effects.



(a) Optimal Regulation

Figure 2: Optimal regulation (from Jackson and Pernoud, 2021)

The graph in **Figure 2** shows that bailouts are the optimal choice when both the expected excess returns from investing in risky assets and bank i's centrality in the financial network are high. When the risk premium is high, it is desirable to maximize returns by investing in the risky asset, while high centrality indicates that bailouts are preferred to a laissez-faire approach. On the other hand, when the returns are lower, but the centrality remains high, it is better to impose portfolio restrictions rather than a laissez-faire approach. If the centrality is low enough, but the returns are high, then laissez-faire becomes the optimal choice. To summarize:

- laissez-faire is best when centrality is low and excess returns are relatively high;
- portfolio restrictions become optimal when excess returns are low and centrality is relatively high; and
- bailouts are optimal when excess returns are high and so is centrality.

To apply this model, it is necessary to first provide certain definitions on Financial Networks and on how to establish the network on the banks.

2.2 Financial Networks

Financial networks are distinct in that they exhibit a significant level of interconnectedness and interdependence. A complex web of connections exists between financial institutions and markets, which include direct financial relationships, indirect exposures through shared counterparties, as well as informational and reputational ties (Battiston et al., 2012; Upper and Worms, 2004).

The high level of interconnectedness and interdependence in financial networks renders them highly vulnerable to systemic risk. A disturbance or shock to one area of the network can propagate rapidly to other areas, potentially resulting in a chain reaction of failures and a wider systemic crisis (Elliott et al., 2014).

Financial networks also possess a critical characteristic of complexity. These networks are defined by a vast number of nodes and links, which complicates the process of identifying and comprehending the fundamental dynamics and vulnerabilities of the system. Furthermore, this complexity poses challenges to the implementation of sound regulatory policies and effective risk management practices.

Lastly, financial networks exhibit a characteristic of constant evolution and adaptation to changes in market conditions and economic environments. With the continuous development of new financial products and technologies, financial institutions strive to optimize their returns and risk management practices. Consequently, financial networks are incredibly dynamic, requiring constant monitoring and analysis to detect emerging risks and vulnerabilities.

In summary, financial networks are distinct and unique, posing considerable challenges and complexities that require careful examination. Nevertheless, studying and managing systemic risk in the financial system necessitates a comprehensive understanding of financial networks.

2.2.1 What is special about Financial Networks?

In contrast to other networks, such as those involving contagious diseases, financial networks can exhibit non-monotonicity (Jackson and Pernoud, 2021). This phenomenon describes situations where the introduction or elimination of a node or link in the network can produce unforeseen alterations in the system's overall stability or risk. This implies that the association between network structure and systemic risk is not always straightforward or monotonic. The inclusion or exclusion of specific nodes or links can have non-linear or even counterintuitive implications on the system's overall risk.

One example of non-monotonicity in financial networks is the "too interconnected to fail" phenomenon, where increasing interconnectedness between financial institutions can initially lead to greater efficiency and diversification of risk, but eventually lead to increased systemic risk if the network becomes too tightly connected. This can occur due to the risk of contagion, where the failure of one institution can lead to a chain reaction of defaults and contagion throughout the network.

Another example is the "robust yet fragile" phenomenon, where networks that are highly robust to small shocks or disturbances can become extremely fragile and prone to systemic risk in the face of larger shocks or crises. This can occur due to the presence of critical nodes or links in the network, which may be highly connected and thus initially increase the robustness of the network but become points of vulnerability in the face of larger shocks.

Overall, non-monotonicity is an important phenomenon to consider when analyzing and managing systemic risk in financial networks, as it highlights the need to take a nuanced and dynamic approach to risk management that accounts for the complex and often counterintuitive relationships between network structure and systemic risk.

2.2.2 Defining Financial Networks

Typically, we have information on the assets and liabilities of each bank in the interbank market, but not on the specific interconnections between banks. In such cases, estimating the interbank network is necessary when performing contagion simulations or evaluating other risk metrics.

Two widely used approaches for estimating the interbank network are the maximum entropy method (Upper and Worms, 2004) and the minimum density estimation method (Anand et al, 2015).

The maximum entropy estimation method is based on using information about assets and liabilities to construct an estimate of the interbank network. The estimate assumes that each bank strives to distribute its exposures as evenly as possible, given the restrictions.
Although this method may be effective in certain instances, it does not fully replicate the properties of interbank networks, which are typically sparse and disassortative.

An alternative to the maximum entropy method is the "minimum density" estimation method proposed by Anand et al (2015). This method takes into account the sparsity of interbank networks by estimating a network with the fewest possible links that still satisfy a set of predetermined constraints. By doing so, it produces a more accurate representation of the actual interbank network and can improve the accuracy of contagion simulations and other risk assessments.

For this estimation I will use the package “NetworkRiskMeasures” in R created by Carlos Cinelli and Thiago Cristiano Silva.

2.3 Important definitions and equations

In this section I will present some relevant definitions and equations. Most of them are based on the papers of Jackson and Pernoud (2019 and 2021) with some adaptations.

The value of a bank (book) is given by:

$$V_i = \pi_i + \sum_j (d_{ij}(V) - D_{ji}) \tag{1}$$

where π_i represents the value of bank i's portfolio of investments out of the financial system and d_{ij} is the value that bank j owns to bank i. Here, for simplicity, it is assumed limited liability and equalized priority of all debt. Therefore:

$$d_{ij}(V) = min\left\{ D_{ij}, \frac{D_{ij}}{\sum_k D_{kj}} \left[\pi_j + \sum_k d_{jk}(V) \right] \right\} \tag{2}$$

In another words, the value of the debt d_{ij} is the minimum between the actual value of the debt (D_{ij}) and the fraction available of bank's j value to pay this debt.

A bank is said to default when the value of assets does not cover liabilities. In this case bankruptcy costs will incur as follows:

$$b_i(V,p) = \begin{cases} 0 & if \ ASSETS + RETURN \geq LIABILITIES \\ \beta_i(V,p) & if \ ASSETS + RETURN < 0 \end{cases} \tag{3}$$

Therefore the bankruptcy cost depends on the the health of other banks as well as the value of the primitive investments. It it is discontinuous as said before, being zero if the assets plus the return on investments surplus the liabilities. Otherwise the value is $\beta_i(V,p)$ and corresponds to:

$$\beta_i(V,p) = b + a * (ASSETS + RETURN), \quad with \ \ b \geq 0 \ \ and \ \ 0 < a < 1 \tag{4}$$

where a indicates that in case of default the bank loses a fraction of its value and b represents some additional fixed cost.

Finally, I am defining the bailout costs as:

$$c_i = (ASSETS + RETURN) - LIABILITIES \tag{5}$$

which is the liquidation value of the institution ($ASSETS - LIABILITIES$) and include the returns as a recoverable part of the cost.

Next step is defining the Financial Centrality Measures.

2.4 Financial Centrality Measures

Measuring the contribution of banks to systemic risk can be accomplished by identifying central, significant, or systemic nodes in the network using financial centrality concepts.

The rationale behind this is that if a bank is highly central, it can pose a significant systemic risk. Researchers such as Jackson and Pernoud (2019 and 2021), Belhaj et al. (2020), and Capponi and Chen (2015) have proposed various measures to capture this phenomenon.

In this study, I will first employ traditional centrality measures and subsequently utilize measures that account for the particularities of financial connections and systemic risk.

2.4.1 Traditional centrality measures

A first approach to this problem would be to use traditional centrality measures from network theory.

Here I briefly discuss some of the most commonly used traditional centrality measures:

- Degree centrality: this measures the number of connections or links that an entity has with other entities in the network
- Betweenness centrality: this measures the extent to which an entity lies on the shortest path between other entities in the network.
- Eigenvector centrality: this measures the importance of an entity based on the importance of the entities it is connected to.
- Closeness centrality: measures how close a node is to all other nodes in the network, based on the length of the shortest paths between them.
- Alpha centrality: also known as Katz centrality, is a measure of centrality that takes into account the entire network structure and the importance of all nodes in the network, rather than just the immediate neighbors of a node.

Although these measures are useful for evaluating the centrality of institutions, they may not be directly linked to financial theory, as they may not consider the unique characteristics of financial connections and systemic risk.

2.4.2 Financial Centrality Measures

The "NetworkRiskMeasures" package offers other noteworthy measures, such as the impact susceptibility and impact diffusion. The impact susceptibility assesses the potential contagion paths that can affect a vertex in relation to its direct contagion paths. If the impact susceptibility is greater than 1, this indicates that the vertex is susceptible to contagion from vertices beyond its immediate neighbors (i.e., remotely vulnerable).

On the other hand, the impact diffusion measures the influence that a node has on the propagation of impacts in the network. It quantifies the impact of removing a vertex's power to propagate contagion on the impact susceptibility of other vertices in the network.

Nevertheless, the primary measures that this paper focuses on are those proposed by Jackson and Pernoud (2019) for measuring the systemic importance of a bank.

The authors provide a network-based measure of financial impact of a given organization. Conceptually, given their approach, there is a unique and clear way to assess financial impact. What limits its implementation is a lack of regulation requiring all counterparties to be revealed to a central bank or other oversight agency (that is why I will simulate my own data for this study)

First measure is the Net Financial Centrality (NFC). The NFC of bank i, given a network (D, S), a vector of investments q, and a change in its investment choices from q_i to q_i' can be determined as follows:

$$NFC_i(q, q_i'; D, S) = \mathbb{E}_p\left[\sum_j b_j(V(q), p) - b_j(V(q_{-i}, q_i'), p)\right] \tag{6}$$

Therefore NFC represents the total impact on the economy that comes from a change in i's investment strategy (from q to q'), based on the bankruptcy costs that are incurred. If there are no changes in bankruptcy costs, then the net financial centrality of i is 0.

After that, the second measure is the Bailout Centrality (BC). This measure represents the impact of guaranteeing to bailout a particular bank, as follows:

$$BC_i(q; D, S) = \mathbb{E}_p\left[\sum_j b_j(V, p) - b_j(V_{-i}, V_i^+, p)\right] \tag{7}$$

where V_{-i} is calculated based on the assumption that bank i does not default on any payments and retains its value V_i^+. In another words, it reflects the total impact on bankruptcy costs when a bank is insured by the government and bailed out in the event of insolvency, as opposed to a scenario where it is left to fail.

2.5 Macroprudential Regulation, Bailout or Laissez-faire

I employed a model based on the one developed by Jackson and Pernoud (2019) to examine the trade-offs between ex post bailouts, portfolio regulation policies, and laissez-faire approaches in which banks are permitted to invest as they choose and are not bailed out in the event of default.

The idea is that the optimal restriction balances the gain from the risk premium with the overall societal expected bankruptcy costs. Therefore, this problem solves:

$$\max_{q_i} \quad q_i\mathbb{E}[p_i] + (1 - q_i)(1 + r) - \mathbb{E}_p\left[\sum_j b_j(V(q_i, q_{-i}), p)\right] \tag{8}$$

where the first component of the objective function reflects the expected return on the portfolio and increases linearly with q_i due to the inclusion of a risk premium. The second component captures the expected bankruptcy costs, which exhibit a discontinuity at certain values of q_i. This discontinuity arises because making small adjustments to i's investment may cause a shift in the set of defaulting banks under certain conditions.

Due to the presence of these discontinuities, there may be multiple optimal levels of investment that need to be evaluated and compared. One such level is to allow bank i to operate without regulation and choose $q_i^* = 1$. Another optimal level is the critical threshold of investment $\bar{q}_i$ in the risky asset, below which bank i remains solvent regardless of the actual return of the asset. This threshold can be determined by solving the following equation:

$$(1 - \bar{q}_i)(1 + r) = D_i^L \quad or \quad \bar{q}_i = 1 - \frac{D_i^L}{1 + r} \tag{9}$$

Therefore, it is possible to compute the thresholds as discussed in the section 2.1 and illustrated in the **Figure 1**. Such thresholds will allow the optimal decision and are as follows.

It would be preferable to implement regulation rather than a laissez-faire approach if:

$$\mathbb{E}[p_i] - \mathbb{E}\left[\sum_j b_j(V(1, q_{-i}), p)\right] < \left(1 - \frac{D_i^L}{1 + r}\right)\mathbb{E}[p_i] + \left(\frac{D_i^L}{1 + r}\right)(1 + r) \tag{10}$$

In summary, limiting bank i's portfolio to prevent its default (regulating) is advantageous if the cost of such restrictions, represented by the reduction in risk premium necessary to maintain the bank's solvency, is lower than the expected reduction in bankruptcy costs.

The other threshold shows that bailing out bank i whenever it is insolvent is preferred to regulate investments if:

$$\left[\frac{\mathbb{E}[p_i]}{1+r} - 1\right] D_i^L > \mathbb{E}_p\left[\sum_j b_j(V_i(q_{-i}, q_i^*)^+, V_{-i}(q_{-i}, q_i^*), p)\right] + c_i \tag{11}$$

In another words, bailing out bank i is preferred when the cost of doing so is smaller than the lost incurred on restricting the bank investment (regulate).

Finally, these equations could be re-written in terms of NFC and BC as presented in section 2.4.2. The results are as follows:

- If $c_i \geq BC_i(q^*)$ then bailout is never optimal and regulate » laissez-faire if:

$$\left[\frac{\mathbb{E}[p_i]}{1+r} - 1\right] D_i^L \leq NFC_i((q_{-i}, q_i^*), \bar{q}_i) \tag{12}$$

- If $c_i < BC_i(q^*)$ then laissez-faire is never optimal and bailout » regulate if:

$$\left[\frac{\mathbb{E}[p_i]}{1+r} - 1\right] D_i^L + BC_i(q_{-i}, q_i^*) - c_i \leq NFC_i((q_{-i}, q_i^*), \bar{q}_i) \tag{13}$$

The concepts of centrality described above enable to determine the most effective method of regulating a bank.

The code for applying this process can be checked in the Appendix (Part II - R code for defining Financial Centrality Measures).

2.6 Stress tests and contagion effects

Both the traditional metrics and the financial centrality measures depend on network topology rather then on a specific shock. Therefore, another way to measure the systemic importance of a bank is to answer the following question: how would the default of the entity impact the system?

A comprehensive understanding of the network is crucial in addressing systemic risk. Accordingly to Jackson and Pernoud (2021) stress testing is a key component in assessing risk, but it typically relies on decentralized balance sheet data, which may not provide complete information on counterparties and the network structure. This local data can miss critical information about which banks are most likely to trigger or be affected by a default cascade.

At this point, my focus is on the impact susceptibility and impact diffusion measures, which account for sequences of successive defaults. If a bank possesses sufficient value and/or their debt liabilities are smaller compared to their assets, they can prevent such chains from occurring. Additionally, it is worth noting that these "chains" can impact some banks multiple times and intersect with each other.

I make use of the contagion() function, which is available in the "NetworkRiskMeasures" package, to simulate a contagion process within the network. This function takes into account the exposure matrix, capital buffer, and node weights of the Banks as its input parameters. It also provides the flexibility to select from different propagation methods or even create a custom one. Currently, I am utilizing two distinct approaches for propagation: the conventional default cascade and the DebtRank.

The DebtRank approach, as proposed by Bardoscia et al (2015), assumes a linear shock propagation, which means that if a bank experiences a 10% loss in its capital buffer, it transmits losses of 10% of its debts to its creditors.

However, it is possible to consider an alternative propagation method. For instance, a bank may not transmit contagion unless it defaults. In such cases, the contagion method can be changed to a threshold-based approach.

In addition to the aforementioned use cases, I also employed the contagion() function to simulate arbitrary contagion scenarios. For instance, I investigated the impact of simultaneous stress shocks ranging from 1% up to 10% across all banks on the entire system.

These implementations can be seen in Appendix (Part III - R code for computing Stress Tests and Contagion Effects).

2.7 Data generating

Due to the difficulty of accessing all the required data for this paper, I will be simulating/generating the necessary data at this moment. Initially, I have generated random values for assets and liabilities. It is worth noting that I took care to generate heavy-tailed values using a lognormal distribution, as these values are typically fat-tailed in actual data.

To simplify matters, I am assuming that all connections (assets and liabilities) in this work are interbank, meaning that there are no investments made outside of the financial system (i.e., $\pi_i = 0$ in equation (1)). Additionally, it is important to note that I have maintained the total sum of values of assets and liabilities as equal by using proportions. Furthermore, I have generated capital buffers as a function of the liabilities. Using these values in conjunction with the assets, I have estimated the balance sheet for each bank, which will serve as weights in the nodes, thereby providing some economic significance to the measures.

Regarding the returns, I am considering two types of investments, each consisting of 5 periods (although this value is arbitrary and could be adjusted). First, I am considering a risk-free investment, where the returns are sufficient to cover - at least - the total liabilities (such that if the bank invests solely in the risk-free investment, it will not become insolvent). On the other hand, I have generated random returns for a risky investment, with a mean equal to that of the risk-free investment, but with values fluctuating to mimic the returns of a risky investment that could be either greater or smaller than the risk-free investment.

Lastly, I have generated random values for $0 < q < 1$, $q < q' < 1$, $0 < a < 1$, and $b > 0$. These values correspond respectively to the share of investment in the risky asset, a change in the investment strategy, the share of assets lost in case of bankruptcy, and a fixed cost in case of bankruptcy. These values will be crucial in estimating the bankruptcy cost (as in equation (4)), as well as estimating the NFC and BC (equations (6) and (7)). Therefore, they are essential for computing the thresholds and defining the optimal regulatory decisions.

I have encapsulated all of these procedures into a single function that is capable of generating this data for any desired number of institutions. For the purpose of this work, I have used 100 institutions. You can refer to the code in the Appendix (Part I - R code for generating the dataset) for more information.

In addition to generating the dataset, I have also calculated the financial centrality measures using the generated data. To do so, I utilized minimum density estimation to construct the adjacency matrix. Using this matrix, apart from computing the traditional centrality measures I managed to create the variables required to calculate the other financial centrality measures, particularly NFC and BC.

Therefore, I have computed the returns on investments made by each bank, considering whether they invested in risky or risk-free assets. Using these values, I have determined the impact of potential defaults in case a bank fails to generate enough returns to cover its liabilities. I have done this by utilizing matrices to store the weights and values of such debts, if any, and then subtracting them from the original returns. Subsequently, I have calculated the bankruptcy and bailout costs in a regular scenario, accounting for the discontinuity as specified in equation (3).

However, I had to simulate two distinct situations to calculate the NFC and BC measures.

Initially, I computed the maximum value of $\bar{q}_i$, as specified in equation (9), which represents the maximum amount that banks could invest in the risky asset, ensuring that they have enough investment in the risk-free asset to pay back their liabilities. This value represents the regulatory measure.

Using this $\bar{q}_i$ value, I had to simulate, using loops, the impact of regulating each bank, i.e., investing $q_i = \bar{q}_i$, on its bankruptcy cost. For this computation, I had to consider every possible case once each bank's failure affects all the other banks in the system differently, which required looping through each bank.

After performing these computations, I could determine the NFC for each bank as the difference in the total bankruptcy costs in the scenarios where this bank changes investment or not.

Similarly, in order to compute BC, I simulated scenarios where each bank's debt is cancelled one at a time. By creating a matrix that cancels all possible debts of each bank in each round, this allows me to see the effect of a bailout of each bank. Using loops to simulate the cancellation of debts for each bank in each round, and iterating through all banks, I computed the BC for each institution.

You can find these procedures in the Appendix (Part II - R code for defining Financial Centrality Measures).

3 Results

In this section I present my preliminary results.
I start by showing the results of the estimated Financial Network created for the simulation on the financial institutions data. After that, I present the results of the main model, i.e. what is the optimal regulation for the banks simulated. In this part I also show the results comparing all the Financial Centrality Measures computed. Finally, in the last part I present the results of stress tests simulations and how the contagious effect affect the institutions.

3.1 Financial Network

As said before, usually one do not observe the real network, only the marginals (total assets and liabilities). Thus, it is necessary to estimate the adjacency matrix before running the contagion simulations or calculating other network measures. For this sense, I used the package created by Cinelli and Silva to compute the minimum density estimation as proposed by Anand et al (2015).

This method takes into account the sparsity of interbank networks by estimating a network with the fewest possible links that still satisfy a set of predetermined constraints. By doing so, it produces a more accurate representation of the actual interbank network and can improve the accuracy of contagion simulations and other risk assessments.
In **Figure 3** it is possible to see the estimated interbank network.

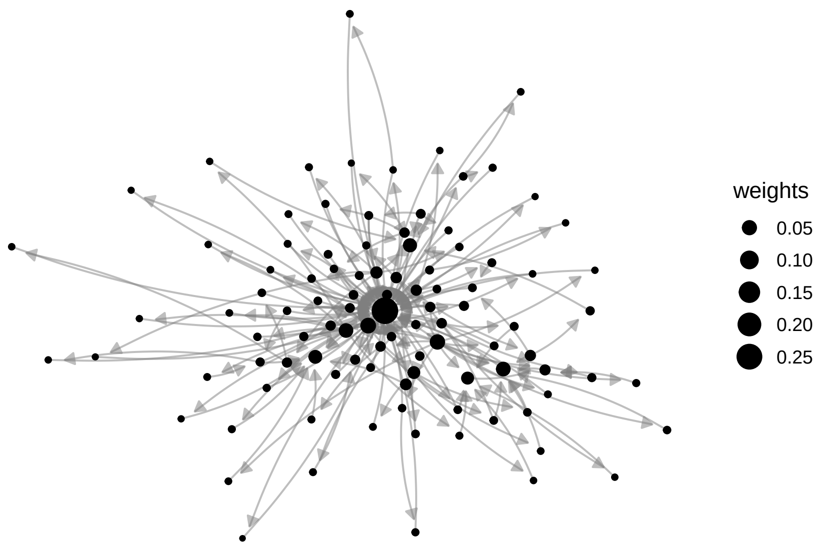


Figure 3: Estimated interbank network

As one can see, the resulting network is sparse and disassortative. In fact, the edge density returned 0.0201 while the assortativity degree returned -0.4422.

3.2 Optimal regulation decisions

After creating the adjacency matrix, the impact of a default in one bank on the others was computed, taking into account the interbank network. Using this information, I computed the NFC and BC measures according to the methodology explained in section 2.4.2. Finally, the optimal regulation decision was determined for the simulated data, i.e. Regulate vs Bailout vs Laissez-faire. The results can be found in **Table 1**.

Table 1: Optimal regulation decisions

Optimal	Frequence
bailout	13
laissez-faire	37
regulate	50

In Table 1, the results show that for the simulated data, the optimal decision would be to bailout 13 financial institutions ex-post, regulate ex-ante 50, and adopt a laissez-faire approach for the remaining 37. As the total number of banks simulated was 100, these values can be interpreted as percentages.

It is worth noting that the optimal regulation decision depends not only on the total number of banks in the system, but also on the specific network structure and the degree of interdependence among the institutions. In particular, a highly interconnected network with many strong links may require more stringent regulations to prevent the spread of contagion in case of default.

Finally, it is also relevant to observe that the optimal decision varies greatly depending on the values of the parameters used in the simulation. For example, different assumptions about the distribution of returns, the level of risk aversion of the banks, and the magnitude of the fixed costs of bankruptcy can lead to different results. Therefore, the analysis of the optimal regulation policy must take into account the specific characteristics of the financial system under consideration.

3.3 Stress Tests and Contagion Effects

In this section, I will be examining the outcomes of simulating contagion processes on the network.

To begin with, I will be scrutinizing the results of the linear propagation approach (DebtRank), which operates on the assumption of a linear shock propagation. This means that if a bank undergoes a 10% loss in its capital buffer, it will transmit losses equivalent to 10% of its debts to its creditors.

The outcomes for the top 10 banks with additional stress in this simulation are presented in **Table 2**.

Table 2: Propagation function: DebtRank (10 first)

Bank	original_stress	additional_stress	original_losses	additional_losses	additional_defaults
64	26.93	50.02	107.99	779.33	66
49	3.59	34.62	23.88	102.60	24
74	4.49	26.55	53.36	95.43	19
25	2.80	23.78	35.60	61.37	16
96	3.88	22.98	53.91	80.68	14
67	0.39	22.69	0.82	22.48	12
39	1.87	21.38	4.53	61.81	10
31	1.44	21.37	17.13	33.12	14
32	0.28	21.23	1.39	15.93	10
57	0.24	21.23	2.50	16.55	10

These results indicate the potential impact of the default of individual banks on the overall financial system under a linear shock propagation assumption.

For instance, let's consider the outcomes for bank 64. Despite representing around 27% of the simulated financial system, its default would result in an additional stress of 50% of the system under linear shock propagation. This additional stress would lead to losses of $ 779.33 (billions of dollars, for example) and cause the default of other 66 financial institutions. Similarly, bank 67, which constitutes only 0.39% of the system, would result in an additional stress of almost 23% of the system or 14 other banks if it defaults. These results suggest that the default of even small banks can have a considerable impact on the stability of the overall financial system.

These results are also showed in **Figure 4** where is possible to see the effects for all banks.

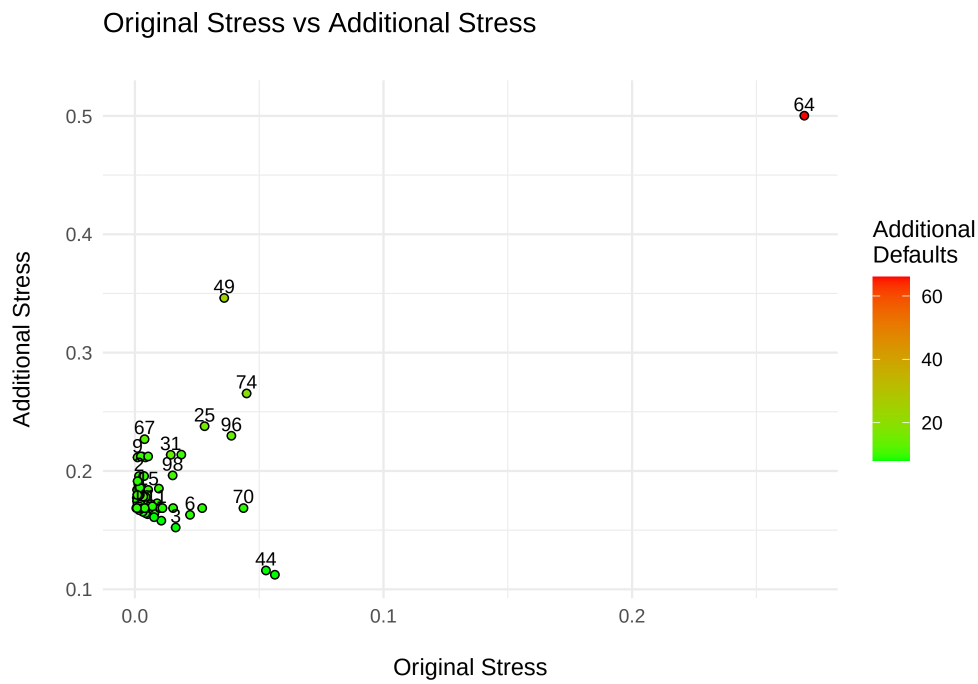


Figure 4: Original Stress vs Additional Stress

It is possible to adopt alternative propagation methods that differ from the linear shock propagation assumption. For instance, one alternative is to utilize a threshold-based approach, where a bank will not transmit contagion unless it defaults.

The outcomes for the top 10 banks with additional stress in this simulation are presented in **Table 3**.

Table 3: Propagation function: Threshold (10 first)

Bank	original_stress	additional_stress	original_losses	additional_losses	additional_defaults
64	26.93	47.94	107.99	722.38	61
49	3.59	24.56	23.88	82.18	16
74	4.49	22.28	53.36	86.03	15
96	3.88	20.70	53.91	77.06	13
15	0.53	16.93	0.78	12.28	9
28	0.22	16.18	0.45	12.87	9
69	0.78	15.40	0.80	11.70	8
87	1.02	15.40	1.18	18.09	8
56	1.54	15.40	0.51	9.46	8
39	1.87	14.86	4.53	50.76	4

While the top 3 banks with the highest additional stress may remain consistent with the DebtRank approach, several other banks may show varying results under the threshold-based approach, suggesting that the two methods generate different outcomes.

This highlights the importance of selecting an appropriate propagation method that aligns with the specific characteristics of the financial system being analyzed, and the type of shock being studied. Furthermore, using multiple propagation methods can provide a more comprehensive and robust analysis of the potential contagion risk within the financial system.

Apart from the aforementioned scenarios, I conducted simulations of arbitrary contagion scenarios. In this particular case, I examined the impact of simultaneous stress shocks ranging from 1% up to 10% on all banks in the system.

The results of this simulation can be seen in **Table 4** and **Figure 5**.

Table 4: Arbitrary contagion scenarios

scenario	additional_stress	original_losses	additional_losses	additional_defaults
10 pct shock	41.77	36.79	173.27	46
9 pct shock	41.76	33.11	159.81	45
8 pct shock	41.72	29.44	146.18	45
7 pct shock	41.66	25.76	132.49	44
6 pct shock	41.60	22.08	118.78	44
5 pct shock	41.49	18.40	104.76	42
4 pct shock	41.21	14.72	89.57	41
3 pct shock	40.85	11.04	74.24	38
2 pct shock	39.74	7.36	57.82	31
1 pct shock	37.34	3.68	39.63	26

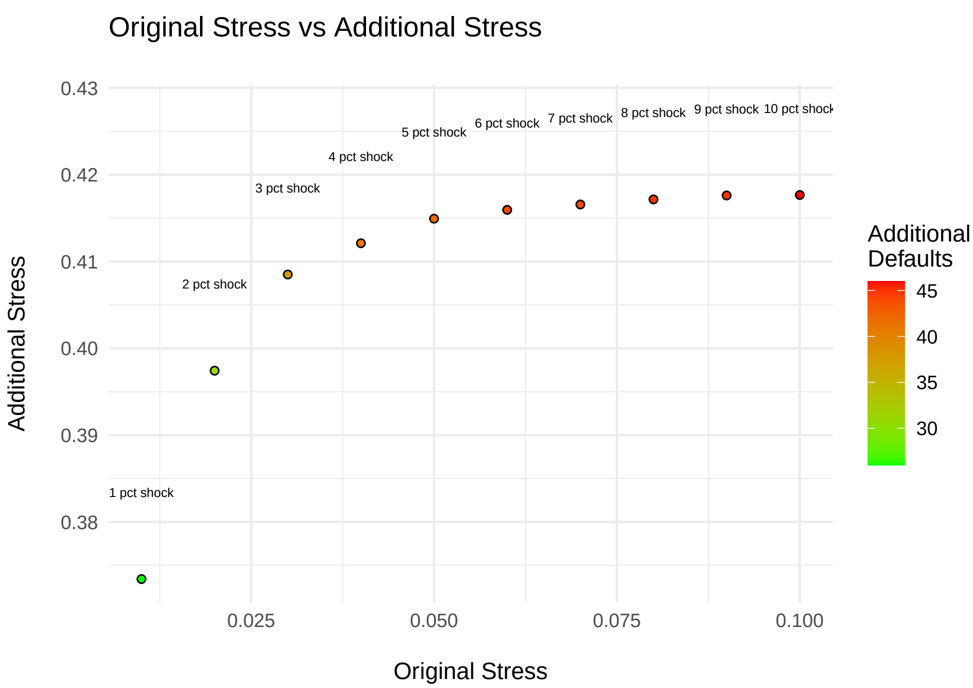


Figure 5: Arbitrary contagion scenarios

In this example, a 10% shock in all banks results in an additional stress of 42% in the system, which is a four-fold amplification of the initial shock.

This outcome suggests that the financial system may be highly vulnerable to contagion effects, particularly under severe and simultaneous stress shocks. Such scenarios can trigger a cascade of defaults and amplify losses, leading to significant disruptions in the financial system and potentially spilling over to the wider economy.

It's important to highlight the fact that these results might not be interpreted too literally and used as the sole determinant of the systemic importance of a financial institution. The additional stress indicator, such as the DebtRank, can serve as another measure of the institution's systemic importance, but it should be considered in conjunction with other factors, such as the institution's size, interconnectedness, and criticality to the overall functioning of the financial system.

To provide a more comprehensive assessment, I computed all these methods together and compared them in **Table 5**.

Table 5: Ranking

	DebtRank	cascade	degree	eigen	impd	total_assets	total_liabilities	buffer	BC	NFC
64	1	1	1	1	1	87	1	1	1	2
49	2	2	5	11	6	24	3	5	3	1
74	3	3	7	23	5	85	2	3	6	4
25	4	10	8	15	20	80	6	4	4	6
96	5	4	8	34	22	79	4	2	2	8
67	6	25	11	31	41	33	20	27	10	12
39	7	8	11	36	23	66	5	10	8	3
31	8	16	9	43	15	35	7	6	9	10
32	9	15	12	49	21	65	26	20	10	12
57	9	15	12	54	21	97	23	14	10	11

In **Table 5**, I have presented all the centrality measures computed and ranked by the DebtRank. Although some results are convergent, such as the fact that bank 64 is ranked first in 8 out of 10 measures, it is crucial to note that it is not ranked first in a very important measure, which is the NFC. This fact underscores the significance of combining different measures. By combining multiple measures of systemic importance, one can obtain a more accurate and comprehensive perspective of the potential risks that individual financial institutions pose to the stability of the financial system.

4 Conclusion and Future Research

This paper examines the relationship between **Financial Networks** and **Systemic Risk** by analyzing the optimal regulation decisions of ex-ante regulation, ex-post bailouts, or laissez-faire approaches. The relevance of the study lies in highlighting the importance of systemic risk in the present-day context, owing to the increasing interconnectedness and complexity of financial systems, as well as the occurrence of significant events such as the global financial crisis, COVID-19 pandemic, and SVB crises.

A methodology proposed by Jackson and Pernoud (2019) is applied to compute the optimal regulation decisions using simulated data. The paper also compares various Financial Centrality Measures while assessing contagion effects through stress simulations.

In addition to the findings presented in this paper, my primary focus for future endeavors is to obtain relevant financial data from institutions to apply the methodology and measures computed in this study to real-world scenarios. This will enable me to analyze past financial crises such as the 2008 crisis and determine if regulators in the US or Europe acted in accordance with the theories expressed in the models used in this study.

In addition to that, to further improve the study I intend to incorporate NFC and BC metrics in the stress tests and simulations for contagious effects. Moreover, I aim to generate data simulations with different distributions to to evaluate their impact on the results.

Finally, it is important to note that this is a work in progress, and many definitions and the code require further revisions and optimization. Nevertheless, I aim to conduct sensitivity analysis to determine the robustness of the results and investigate the effects of varying parameters on the outcomes. Overall, these future endeavors are expected to contribute to the advancement of systemic risk analysis and the development of effective policy-making measures.

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Appendix

Part I - R code for generating the data set

```
# Generate data
# Description: simulate data on financial institutions
# e.g. assets, liabilities, buffer, BS and CF
# Author: Wagner Eduardo Schuster (2023/03)

# cleaning
rm(list=ls());gc()
# setting work directory
setwd("~/QEM/BARCELONA/Networks/Paper")

# function to generate data (n = number of banks)
generate_financial_data <- function(n) {
  # Set seed for reproducibility
  set.seed(123)

  # Generate random data for financial institutions
  ## Assets (fat-tailed using lognormal dist)
  tot_assets <- rlnorm(n, 0, 2)

  # fat-tailed liabilities
  tot_liabilities <- rlnorm(n, 0, 2)

  # guaranteeing tot_assets = tot_liabilities
  tot_assets <- sum(tot_liabilities) * (tot_assets/sum(tot_assets))

  # Buffer (capital) as a function of liabilities
  buffer <- runif(length(tot_liabilities))*tot_liabilities

  # Weights (balance sheet) as a function of assets, buffer and liabilities
  weights <- (tot_assets + tot_liabilities + buffer + 1) + rlnorm(n, 0, 1)

  # Cash flow (5 years)
  ## risk-free
  library(FinCal)
  ### interest rate
  ## try with positive return...
  ## (at least pay back your liabilities + random positive return)
  r_riskfree <- 0.05 * runif(n,1,1.02)

  ### calculate the fixed payment amount using the pmt() function
  cf_riskfree <- -pmt(r_riskfree, 5, tot_liabilities,0)

  ## risky
  cf_year1 <- rnorm(n, mean = cf_riskfree, sd = cf_riskfree/5)
  cf_year2 <- rnorm(n, mean = cf_riskfree, sd = cf_riskfree/5)
  cf_year3 <- rnorm(n, mean = cf_riskfree, sd = cf_riskfree/5)
  cf_year4 <- rnorm(n, mean = cf_riskfree, sd = cf_riskfree/5)
  cf_year5 <- rnorm(n, mean = cf_riskfree, sd = cf_riskfree/5)
```

```r
  # share invested in risky assets (q)
  q <- runif(n, 0, 1)
  # random change in share invested in risky assets (q)
  # Q_PRIME >= Q
  q_prime <- runif(n, q, 1)

  # share assets lost in case bankruptcy (a)
  # 0 < a < 1
  a <- runif(n, 0, 1)

  # fixed cost in case bankruptcy (b)
  # b > 0
  b <- rlnorm(n, 0, 1)

  # Create data frame
  data <- data.frame(
    institution_id = 1:n,
    total_assets = tot_assets,
    total_liabilities = tot_liabilities,
    buffer = buffer,
    weights = weights,
    cf_riskfree = cf_riskfree,
    cf_year1 = cf_year1,
    cf_year2 = cf_year2,
    cf_year3 = cf_year3,
    cf_year4 = cf_year4,
    cf_year5 = cf_year5,
    q = q,
    a = a,
    b = b,
    q_prime = q_prime
  )

  # Return data
  return(data)
}

data <- generate_financial_data(100)

# save data
write.csv2(data,"Data/data.csv")
```

Part II - R code for defining Financial Centrality Measures

```r
# Financial centrality measures
# Description: compute financial centrality measures
# e.g. NFC and BC. also the standard ones: eigen, degree, closeness, etc
# Author: Wagner Eduardo Schuster (2023/03)

# cleaning
rm(list=ls());gc()
```

```r
# setting work directory
setwd("~/QEM/BARCELONA/Networks/Paper")

# load data
data <- read.csv2("Data/data.csv")
data <- data[,-1]

# Loads the package
library(NetworkRiskMeasures)
# Minimum Density Estimation
set.seed(192) # seed for reproducibility
MD <- matrix_estimation(rowsums = data$total_assets, colsums = data$total_liabilities
                        , method = "md")

# rownames and colnames for the matrix
rownames(MD) <- colnames(MD) <- data$institution_id

# graph
library(ggplot2)
library(ggnetwork)
library(igraph)

# converting  network to igraph object
gmd <- graph_from_adjacency_matrix(MD, weighted = T)

# adding other node attributes to the network
V(gmd)$buffer <- data$buffer
V(gmd)$weights <- data$weights/sum(data$weights)
V(gmd)$assets  <- data$total_assets
V(gmd)$liabilities <- data$total_liabilities

# ploting with ggplot and ggnetwork
set.seed(20)
netdf <- ggnetwork(gmd)

ggplot(netdf, aes(x = x, y = y, xend = xend, yend = yend)) +
  geom_edges(arrow = arrow(length = unit(6, "pt"), type = "closed"),
             color = "grey50", curvature = 0.1, alpha = 0.5) +
  geom_nodes(aes(size = weights)) +
  ggtitle("Estimated interbank network") +
  theme_blank()

# network density
edge_density(gmd)

# assortativity
assortativity_degree(gmd)

#  Traditional centrality measures, impact susceptibility and impact diffusion
data$degree <- igraph::degree(gmd)
data$btw    <- igraph::betweenness(gmd)
data$close  <- igraph::closeness(gmd)
data$eigen  <- igraph::eigen_centrality(gmd)$vector
```

```r
data$alpha  <- igraph::alpha_centrality(gmd, alpha = 0.5)

# The impact susceptibility and impact diffusion
data$imps <- impact_susceptibility(exposures = gmd, buffer = data$buffer)
data$impd <- impact_diffusion(exposures = gmd, buffer = data$buffer, weights = data$weights)$total

# NPV (risk-free and risky) considering no bank default
library(FinancialMath)
NPV_riskfree <- NA
NPV_risky <- NA
for (bank in 1:length(data$institution_id)){
  NPV_riskfree[bank] <- NPV(data$total_liabilities[bank],c(rep(data$cf_riskfree[bank],5)),1:5,i=0.05)
  NPV_risky[bank] <- NPV(data$total_liabilities[bank],as.numeric(data[bank,7:11]),1:5,i=0.05)
}

## add to the data
data$NPV_riskfree <- NPV_riskfree
data$NPV_risky <- NPV_risky

## total NPV
data$NPV <- data$NPV_risky*data$q + data$NPV_riskfree*(1-data$q)

# APPLY DEFAULT OF OTHER BANKS IN THE NPV
data$debt <- data$total_assets + data$NPV - data$total_liabilities
## keep only negative values (positive are not debt) and turn positive
data$debt <- ifelse(data$debt>0,0,-data$debt)

# create matrix of weights (for liabilities: all columns will sum up to 1)
# proportion the liabilities of bank i are divided among all other banks
W_MD <- MD
for (c in 1:ncol(MD)){
  for (r in 1:nrow(MD)){
    W_MD[r,c] <- MD[r,c] / sum(MD[,c])
  }
}

# now if there is a debt it will be spread accordingly to these weights...
matrix_debt <- MD
for (c in 1:ncol(MD)){
  for (r in 1:nrow(MD)){
    matrix_debt[r,c] <- W_MD[r,c] * data$debt[c]
  }
}

# sum up values of each columns
total <- rowSums(matrix_debt)
matrix_debt <- cbind(matrix_debt,total)

# take out from NPV (banks will not receive if default)
matrix_debt <- as.data.frame(matrix_debt)
data$NPV_new <- data$NPV - matrix_debt$total

# BAILOUT COST
```

```r
## Calculate the liquidation value of the institution
data$liquidation_value <- data$total_assets - data$total_liabilities
## Calculate the estimated bailout cost (account for part recoverable bailout)
data$bailout_cost <- data$liquidation_value + data$NPV_new
data$bailout_cost <- -data$bailout_cost

# BANKRUPTCY COST
## bankruptcy cost would be lose fraction a of all the assets + "b" (additional fixed cost)
## need also to be a function of NPV (for the q)
data$bankruptcy_costs <- data$b + data$a * (data$total_assets + data$NPV_new)

# discontinous function (see Pernoud p. 6)
data$bankruptcy_costs <- ifelse(data$NPV_new + data$liquidation_value<0,
                                data$bankruptcy_costs,0)

# LOOPS FOR NFC AND BC
# create  bankruptcy_prime to collect all these values
bankruptcy_prime <- as.data.frame(matrix(NA, nrow(data), nrow(data)))
for (i in 1:nrow(data)){
  ## NPV prime
  ### same for j (all banks but i)
  data$NPV_prime <- data$NPV
  ### NPV_prime for bank i
  data$NPV_prime[i] <- ( data$NPV_risky[i]*data$q_prime[i]
                       + data$NPV_riskfree[i]*(1-data$q_prime[i]) )

  ## APPLY DEFAULT OF OTHER BANKS IN THE NPV
  data$debt_prime <- data$total_assets + data$NPV_prime - data$total_liabilities
  ## keep only negative values (positive are not debt) and turn positive
  data$debt_prime <- ifelse(data$debt_prime>0,0,-data$debt_prime)

  # use same matrix of weights (W_MD)
  # now if there is a debt it will be spread accordingly to these weights...
  matrix_debt_prime <- MD
  for (c in 1:ncol(MD)){
    for (r in 1:nrow(MD)){
      matrix_debt_prime[r,c] <- W_MD[r,c] * data$debt_prime[c]
    }
  }

  # sum up values of each columns
  total_prime <- rowSums(matrix_debt_prime)
  matrix_debt_prime <- cbind(matrix_debt_prime,total_prime)

  # take out positive values (no impact) and change sign others
  matrix_debt_prime <- as.data.frame(matrix_debt_prime)
  # take out from NPV
  data$NPV_prime_new <- data$NPV_prime - matrix_debt_prime$total_prime

  # bankruptcy cost
  # need also to be a function of NPV (for the q)
  bankruptcy_prime[,i] <- ( data$b + data$a *
                           (data$total_assets + data$NPV_prime_new) )
```

```
  # discontinous function (see Pernoud pag 6)
  bankruptcy_prime[,i] <- ifelse(data$NPV_prime_new + data$liquidation_value<0
                                 ,bankruptcy_prime[,i], 0)
}

# NFC - Net Financial Centrality
for (i in 1:nrow(data)){
  data$NFC[i] <- ( (sum(data$bankruptcy_costs) - data$bankruptcy_costs[i])
                  -(sum(bankruptcy_prime[,i]) - bankruptcy_prime[i,i]) )
}


# LOOP: take out effects of debt in bank i
bailout <- as.data.frame(matrix(NA, nrow(data), nrow(data)))
# create  bailout to collect all these values
# it will receive the bankruptcy cost considering effects on bailout
for (i in 1:nrow(data)){
  # now if there is a debt it will be spread accordingly to these weights...
  # need to spread but taking out the effects of debts from bank i
  matrix_debt_bailout <- MD
  for (c in 1:ncol(MD)){
    for (r in 1:nrow(MD)){
      matrix_debt_bailout[r,c] <- W_MD[r,c] * data$debt[c]
    }
  }

  # erase the debt of bank i (like bailout it...)
  matrix_debt_bailout[,i] <- 0
  total_bailout <- rowSums(matrix_debt_bailout)

  matrix_debt_bailout <- cbind(matrix_debt_bailout,total_bailout)

  # take out from NPV
  matrix_debt_bailout <- as.data.frame(matrix_debt_bailout)
  data$NPV_bailout <- data$NPV - matrix_debt_bailout$total_bailout

  # bankruptcy cost
  # need also to be a function of NPV (for the q)
  bailout[,i] <- ( data$b + data$a *
                  (data$total_assets + data$NPV_bailout) )
  # discontinous function (see Pernoud pag 6)
  bailout[,i] <- ifelse(data$NPV_bailout + data$liquidation_value<0
                        ,bailout[,i], 0)
}

# BC - Bailout Centrality
for (i in 1:nrow(data)){
  data$BC[i] <- ( (sum(data$bankruptcy_costs) - data$bankruptcy_costs[i])
                - (sum(bailout[,i]) - bailout[i,i]) )
}


# define q_bar
```

```r
# formula: 1 - (D / (1+r)) i.e. debt / return_risk_free
data$q_bar <- 1 - data$total_liabilities / (data$total_liabilities + data$NPV_riskfree)

# then compute NFC between q and q_bar. but instead of q, is (q_{-i},1). Everybody else
# invest q but bank i invest 1

# LOOP
# create  bankruptcy_prime to collect all these values
bankruptcy_bar <- as.data.frame(matrix(NA, nrow(data), nrow(data)))
for (i in 1:nrow(data)){
  ## NPV bar
  ### same for j (all banks but i)
  data$NPV_bar <- data$NPV
  ### NPV_bar for bank i
  data$NPV_bar[i] <- (data$NPV_risky[i]*data$q_bar[i]
                   + data$NPV_riskfree[i]*(1-data$q_bar[i]) )

  ## APPLY DEFAULT OF OTHER BANKS IN THE NPV
  data$debt_bar <- data$total_assets + data$NPV_bar - data$total_liabilities
  ## keep only negative values (positive are not debt) and turn positive
  data$debt_bar <- ifelse(data$debt_bar>0,0,-data$debt_bar)

  # use same matrix of weights (W_MD)
  # now if there is a debt it will be spread accordingly to these weights...
  matrix_debt_bar <- MD
  for (c in 1:ncol(MD)){
    for (r in 1:nrow(MD)){
      matrix_debt_bar[r,c] <- W_MD[r,c] * data$debt_bar[c]
    }
  }

  # sum up values of each columns
  total_bar <- rowSums(matrix_debt_bar)
  matrix_debt_bar <- cbind(matrix_debt_bar,total_bar)

  # take out from NPV
  matrix_debt_bar <- as.data.frame(matrix_debt_bar)
  data$NPV_bar_new <- data$NPV_bar - matrix_debt_bar$total_bar

  # bankruptcy cost
  # need also to be a function of NPV (for the q)
  bankruptcy_bar[,i] <- ( data$b + data$a *
                         (data$total_assets + data$NPV_bar_new) )
  # discontinous function (see Pernoud p. 6)
  bankruptcy_bar[,i] <- ifelse(data$NPV_bar_new + data$liquidation_value<0
                                 ,bankruptcy_bar[,i], 0)
}

# NFC - Net Financial Centrality
for (i in 1:nrow(data)){
  #  data$NFC[i] <- -sum(data$bankruptcy_costs) - sum(bankruptcy_prime[,i])
  data$NFC_bar[i] <- ( (sum(data$bankruptcy_costs) - data$bankruptcy_costs[i])
                   -(sum(bankruptcy_bar[,i]) - bankruptcy_bar[i,i]) )
```

```
}

# define thresholds
data$optimal <- ifelse(data$bailout_cost >= data$BC
       ,ifelse( (data$NPV_risky/data$NPV_riskfree - 1)*data$total_liabilities
                <= data$NFC_bar, "regulate", "laissez-faire"), "" )

data$optimal <- ifelse(data$bailout_cost < data$BC
       ,ifelse( (data$NPV_risky/data$NPV_riskfree - 1)*data$total_liabilities
                + data$NFC_bar - data$bailout_cost <= data$NFC_bar
                , "regulate", "bailout"), data$optimal )

# count
table(data$optimal)

# save tables
write.csv2(data, "Data/data_fin_cent.csv")
write.csv2(MD, "Data/MD.csv")
```

Part III - R code for compute Stress Tests and Contagion Effects

```
# Stress
# Description: compute stress measures and contagious
# Author: Wagner Eduardo Schuster (2023/03)

# cleaning
rm(list=ls());gc()
# setting work directory
setwd("~/QEM/BARCELONA/Networks/Paper")

# load data
data <- read.csv2("Data/data_fin_cent.csv")
data <- data[,-1]
MD <- read.csv2("Data/MD.csv")
MD <- MD[,-1]

# DebtRank simulation
contdr <- contagion(exposures = MD, buffer = data$buffer, weights = data$weights,
                    shock = "all", method = "debtrank", verbose = F)
summary(contdr)

contdr_summary <- summary(contdr)
data$DebtRank <- contdr_summary$summary_table$additional_stress

# Traditional default cascades simulation (THRESHOLD)
contthr <-  contagion(exposures = MD, buffer = data$buffer, weights = data$weights,
                      shock = "all", method = "threshold", verbose = F)
summary(contthr)

# save results
contthr_summary <- summary(contthr)
```

```r
data$cascade <- contthr_summary$summary_table$additional_stress

# Rankings
rankings <- data[1] # institution id
# add other measures
rankings <- cbind(rankings, data[
  c("DebtRank","cascade","degree","eigen","impd","total_assets", "total_liabilities"
    ,"buffer","BC","NFC_bar")])
# take abs for BC and NFC
rankings$BC <- abs(rankings$BC)
rankings$NFC <- abs(rankings$NFC_bar)
rankings$NFC_bar <- NULL
# rank values
rankings2 <- as.data.frame(lapply(rankings,function(x) as.numeric(factor(-1*x))))
rankings2 <- rankings2[order(rankings2$DebtRank), ]
head(rankings2, 10)

#And the cross-correlations between the metrics:
cor(rankings[-1])

# Simulating arbitrary contagion scenarios
s <- seq(0.01, 0.10, by = 0.01)
shocks <- lapply(s, function(x) rep(x, nrow(MD)))
names(shocks) <- paste(s*100, "pct shock")

cont <- contagion(exposures = gmd, buffer = data$buffer, shock = shocks, weights = data$weights
                  , method = "debtrank", verbose = F)
summary(cont)

plot(cont, size = 2.2)
```