COST-BASED CAP AND FLOOR MODEL FOR PPP INFRASTRUCTURE RISK ALLOCATION

Abstract

There is increasing reliance on private capital to fund infrastructure through Public-Private Partnerships (PPP), but perceived demand risk may deter investors. To attract participation, governments offer risk-mitigation mechanisms like subsidies, guarantees, or term extensions, which often create fiscal burdens and lack clear design guidelines. Cap and floor (collar) options have traditionally been used in concessions, but threshold levels are typically set arbitrarily based on revenue projections. This paper proposes a cost-based approach to defining these thresholds, improving clarity, transparency, and efficiency in risk allocation. The floor secures minimum revenues to cover project debts, while the cap returns excessive profits to public funds. Using a real options framework, we model these thresholds and present a numerical application. Results show the model supports fair, cost-efficient contract design for PPPs, reducing government liabilities and encouraging private investment. It creates a predictable, balanced structure for risk-sharing that protects public resources while maintaining investor confidence.

Keywords: cap and floor thresholds; infrastructure projects; concessions; optimal cutoff values.

1. Introduction

In the last few decades there has been a worldwide trend towards the use of private capital to fund infrastructure projects in transportation, energy, sanitation and other Public-Private Partnership (PPP) infrastructure projects (Marques et al., 2024). These projects are typically awarded through public auctions where competitors place bids to win the right to build, operate and transfer (BOT) the completed project to the granting authority government after a set number of years. Private investors, however, may shy away from projects they consider excessively risky, leaving the government without suitors for a needed infrastructure asset. To attract investors in this class of projects, governments may provide incentives such as risk mitigating mechanisms, financial subsidies, revenue guarantees or even term extensions that reduce the capital costs required or ensure a minimum level of income for the investor in the project.

Except for term extensions, all these mechanisms involve upfront costs for the government or represent costly future contingent liabilities for the taxpayers if not adequately designed. From 1984 to 1994 Mexico developed 3,600 miles of privately financed toll roads, where minimum traffic volumes were guaranteed by the government. The expected traffic levels never materialized and in the wake of the 1994 devaluation of the peso the government was saddled with a $3.3 billion cost to restructure fifty-two of these highway guarantees (Foote, 1997). South Korea also began an ambitious infrastructure development plan in 1998 where it introduced a revenue guarantee over the life of a project in case demand was lower than forecasted. Over time the cost of these guarantees greatly exceeded expectations and after the cumulative payments handed over to the concessionaires passed the $3 billion mark, the government announced it would do away with these provisions (Kim et al.,2019, Park et al., 2018). Spain has used PPP projects for decades to promote toll highways, but in 2013 nine out of fourteen roads awarded between 1999 and 2006 filed for bankruptcy as the economy entered a recession. Due to government backed revenue and loan guarantees, the government´s liability turned out to be €3.56 billion and the roads were nationalized in 2018 (Baeza Muñoz et al., 2021). This shows that these mechanisms must be carefully structured to minimize costs to the taxpayers and society and to avoid inefficient allocations of resources.

A traditional and common mechanism to limit risks in PPP projects is the Minimum Revenue Guarantee (MRG), which is a contractual agreement between the concessionaire and the government that ensures that the private party receives compensation if the revenue levels fall below a predetermined threshold. This guarantees a minimum level of revenues, safeguarding investors against significant losses. The MRG, which is a revenue-based model, is typically used in conjunction with the Excess Revenue Sharing (ERS) mechanism where revenues above a predetermined threshold are handed over to the government, limiting potential windfall profits. This is equivalent to setting an upper cap (ERS) and a lower floor (MRG) on the project's revenues. This arrangement is also known as the cap and floor model or a collar option strategy, which limits both gains and losses by combining a call option held by the government on the project´s revenues, and a put option held by the concessionaire. This approach aims to strike a balance between providing an incentive for private investments and safeguarding public interests in infrastructure development. Since this model has option-like characteristics, the valuation of this mechanism requires the use of option-pricing methods such as the real options approach (Liu and Cheah, 2009, Marques et al., 2019).

These upper and lower threshold levels are typically set as a percentage of the expected demand. Ideally, in the collar mechanism the lower threshold level should allow the private investor to earn a return that covers its cost of capital, so the floor should guarantee enough revenues to attract the investor. However, determining these optimal threshold levels is a complex task. If the thresholds are set too low, project returns may be severely constrained, rendering the guarantee ineffective in shielding the investor from revenue risk. This could lead to operational disruptions, contract renegotiations, or even litigation. Conversely, if set too high, the threshold may enable the investor to capture windfall profits and transfer the majority of the risk to the public sector, resulting in significant fiscal burdens for the government.

Traditionally, these upper and lower levels have been set as fixed percentages of expected demand (Zheng & Jiang, 2023; Brandão et al., 2012; Brandão & Saraiva, 2008; Marques et al., 2024), which is a practice that lacks a clear analytical basis and may result in inefficient risk allocation. Thus, while this mechanism serves as an effective instrument for enhancing the attractiveness of a project to private investors, it also carries the potential to impose substantial financial liabilities on the government and, by extension, the taxpayers. This underscores the critical importance of developing an analytical procedure to determine the optimal threshold levels ensuring both financial sustainability and effective risk sharing. In addition, the expected future demand or future revenues forecasts associated with a project may be subject to significantly more error, and it may be much easier to estimate capital costs and operating expenses.

This article proposes a cost-based rate of return model to determine appropriate collar upper and lower levels for infrastructure projects. As part of the proposed model, the minimum revenue guaranteed to the concessionaire is set at a level sufficient to cover fixed costs, including OPEX, taxes, and debt service. Conversely, the maximum return is capped at a level consistent with the expected rate of return for an investment of comparable risk, ensuring a balanced and rational allocation of risk and reward. To the best of our knowledge, this paper makes three novel contributions to the literature on infrastructure concessions and PPP design. First, it proposes a cost-based rate of return model to define the cap and floor (collar) thresholds in infrastructure projects, departing from the conventional practice of setting these levels as arbitrary percentages of expected revenues. Second, it introduces an analytical and transparent methodology that anchors the minimum revenue guarantee (floor) in the concessionaire's fixed costs - including OPEX, taxes, and debt service - and sets the maximum return (cap) based on the expected return of an investment with similar risk, ensuring balanced risk allocation and avoiding windfall profits. Third, the model incorporates a realistic assumption by limiting long-term traffic demand to the maximum operational capacity of the infrastructure, addressing a common flaw in traditional projections that assume unlimited exponential growth. Our results demonstrate that the proposed model effectively identifies cap and floor levels that minimize the fiscal burden on the government while preserving adequate incentives for private sector participation. This cost-based approach enhances economic efficiency, promotes a more balanced allocation of risk, and contributes to the development of more predictable, transparent, and financially sustainable PPP contracts.

This article is organized as follows. After this introduction we present a review of the relevant literature in the field, followed by a description of the methodology to calculate the cap and floor levels and the modifications required for its application to this class of projects in Section 3. In Section 4 we present a numerical application and in Section 5 we discuss the results. Finally, we conclude.

2. Literature Review

There is a growing body of literature highlighting the significance of appraising risk-sharing mechanisms within infrastructure initiatives. This emphasis affords governmental agencies the ability to assess the budgetary and fiscal implications of such support and also to establish guarantee thresholds that strike a delicate balance – sufficiently high to sustain project economic viability yet judiciously low so as not to overburden government and societal stakeholders (Brandão & Saraiva, 2008). Also, there has been a trend over the past decade towards formulating accounting standards that delineate the treatment of guarantees pertaining to government entities. This initiative aims to enhance transparency across budgetary documents, fiscal reports, and financial statements (Carmichael, Nguyen & Shen, 2019). The Eurostat approach to PPPs, for example, is designed to determine whether, and at what point according to current Eurostat regulations, the entirety of the capital investment in a given PPP project should be considered as a public expenditure, thus affecting the government deficit/surplus. If so, the full extent of debt issued to fund the investment should be disclosed as government debt (European PPP Expertise Centre, 2010).

The importance of valuing these mechanisms justifies the growing attention given by researchers to this topic, and particularly to the development of models and analytical techniques to evaluate guarantees, mainly using the Real Options Approach (ROA) (Adkins, Paxson, Pereira & Rodrigues, 2019; Shi, An & Chen, 2020; Paxson, Pereira & Rodrigues, 2022). Despite this, there has been limited focus on establishing strike prices for a Minimum Revenue Guarantee (MRG – floor) and an Excess Revenue Sharing (ERS – cap), which directly influence the valuation of such mechanisms.

Previous studies (Brandão and Saraiva, 2008; Ashuri, 2010; Liu et al., 2009) determine arbitrary thresholds on MRG and ERS guarantees based on sensitivity analysis. On the other hand, Carbonara, Constantino and Pellegrino (2014) proposed a model that establishes upper and lower boundaries for a single MRG option by limiting total guarantee exposure over the concession period. They assume that, for the private sector, the guarantee must ensure that the sum of discounted cash flows (revenues minus operating costs) and annual guarantees exceeds the required capital expenditures, reflecting a positive Net Present Value (NPV). This ensures the investment is attractive to private investors without specifying a minimum return threshold. Furthermore, from the public sector's perspective, the guarantee level should be economically sustainable and politically acceptable. The model establishes a ceiling to ensure that assets created through the PPP effectively belong to the public sector, adhering to Eurostat’s guidelines.

Following a similar methodology, Buyukyoran and Gundes (2018) assume that the net guarantee threshold is constrained using basic investment decision rules and Eurostat treatment. However, in addition to the Eurostat statement introduced by Carbonara et al. (2014), this paper incorporates the excess revenues obtained by the public sector in the model. They set the guarantee amount as an objective function, where its lower bound maintains the profitability of a private investor, while its upper bound controls the contingent liability of the public sector. Then, they minimize and maximize this function using these boundaries and determine the optimal pairs of values for the revenue lower limit and upper cap. Jin et al. (2021) propose a synthetic approach for optimizing the concession period and minimum revenue guarantee (MRG) in public–private partnerships. They find there is an inverse relationship between concession length and MRG in which the optimal solution depends on a bargaining model.

Another approach to limiting guarantees involves the introduction of rate of return-based ceilings that trigger revenue-sharing mechanisms. In Chile, any revenue that allows the concessionaire to exceed a 15% rate of return is shared with the government. Liu and Cheah (2009) propose a scenario where the government can claim excess cash flows if the internal rate of return (IRR) surpasses 20%. A similar but distinct method is employed in Korea, where revenue bands based on forecasts are established to limit total guarantees (Irwin, 2007). Under this framework, the government claims any revenues exceeding 110%, 120%, and 130% of forecasted amounts during the first, second, and third five-year periods, respectively. In the Korean Highway Project case discussed by Ashuri et al. (2012), revenues above 110% of the forecast are shared equally between the government and the concessionaire.

Wu et al. (2022) also proposed a model to determine the optimal cap and floor threshold levels for traffic guarantees. Their model is based on the optimal risk allocation between participants of a PPP considering the perspective of lenders and the risk tolerances of the participants. They use an objective function to minimize the sum of the probabilities of the concessionaire's NPV being negative and the probability of the cost of government guarantees being greater than a given budget, subject to a maximum probability of default by the concessionaire.

Shan, Garvin and Kumar (2010) analyze the conceptual framework, applicability, advantages, and unique characteristics of the collar option and determine MRG and ERS thresholds for a zero-cost collar based on an arbitrary minimum acceptable rate of return for the put threshold. In a similar vein, Carbonara and Pellegrino (2018) propose a zero-cost win-win collar model and find that the upper and lower thresholds are almost the same, which would result in a risk-free project. An approach closer to ours is presented by Dutton & Lockwood (2017), who propose a cost-based model specifically designed for international electricity transmission links. This model was adopted by the Office of Gas and Electricity Markets (OFGEM, 2021) in Great Britain to incentivize private developers to invest in interconnector capacity by limiting their exposure to fluctuations in electricity prices and uncertainties in physical volume.

Nonetheless, there is a gap in the literature on a systematic approach to identify clearly fair strike prices or levels for compound MRG (floor) and ERS (cap) options, despite the significant impact of such levels on project and guarantee valuations. To address this issue, we propose a cost-based rate of return model designed to establish appropriate collar upper and lower levels for infrastructure projects. Our model estimates fair levels of MRG and ERS that minimize the cost to the government while assuring an adequate level of support for the concessionaire to undertake the project.

This model offers a novel approach that contrasts with prior work, where cap and floor levels are typically determined based on revenue or traffic projections over the course of the project concession. Unlike these approaches, our model considers the capital structure and debt commitments necessary for the project's progress. To the best of our knowledge, this methodology has not been previously applied to roadways or other transportation infrastructure projects. It provides policymakers with a clear, structured way to determine these threshold levels, an area where current methodologies remain underdeveloped.

3. The Model

Our work builds on the OFGEM (2021) model, which is specifically tailored to electricity transmission lines, where project revenues tend to be relatively stable. This stability stems from the fact that except for peaking plants, assets such as transmission lines and power plants typically operate at or near full capacity, and electricity prices follow a mean-reverting process. Consequently, cap and floor levels in this context are set at constant values over the life of the project. In contrast, our proposed model is more versatile and can be adapted to a wide range of infrastructure projects, with a particular emphasis on PPP projects, where traffic revenues are expected to grow exponentially due to the common practice of building assets with significant initial overcapacity. As a result, cap and floor thresholds in our model are not fixed. Additionally, we adopt annuity payments instead of depreciation when calculating the optimal thresholds, as annuities inherently account for the cost of servicing both equity and debt, providing a more accurate financial representation

An important consideration is that we calculate the cap and floor levels over the full value of the capital investment (CAPEX) rather than only on the equity and debt portions, respectively, for the cap and floor. This approach provides a more comprehensive and realistic assessment of the risks associated with the returns of the total investment, leading to a more accurate and holistic analysis. Our method offers greater transparency and clarity in evaluating the returns of PPP projects, ensuring that significant portions of the investment are not overlooked, and thus avoiding potential distortions in the financial analysis.

The calculations of the cap and floor thresholds are performed in three steps. First, an annuity value over the anticipated life of the project is determined by considering the total investment cost of the project. For the floor, an annuity value for the life of the project is calculated based on the cost of debt. Conversely, for the cap, an annuity value is calculated based on the shareholder's opportunity cost of capital. Second, taxes are calculated. The tax calculation for the floor is done by adjusting downwards the project revenue levels until the IRR is equal to the cost of debt. Similarly, for the cap, the revenues are adjusted upward until the IRR is equal to the shareholder's cost of equity. Third, the annual values of the floor and cap are obtained by summing all the allowances, including operational cost (OPEX) and taxes.

The purpose of the floor is to allow an efficient concessionaire developer to recover their costs, which include debt service obligations, maintenance and operating expenses, and incurred taxes. While the floor threshold level should not be sufficient to create value for the shareholders, it should ensure that the concessionaire is at least able to pay its third-party obligations in order to avoid default. Extending the floor to the equity holders ensures that they will receive at least the cost of debt rate on their investment, even though this return will still be well below the cost of equity. This also helps the concessionaire abide by covenant ratios that may be required by the lenders. An additional percentage point may be granted as a reward if the concessionaire maintains a predetermined service level quality. On the other hand, as a way to mitigate risks to the government, a revenue cap is also set where any returns above the cap level are handed over to the government. This cap is set at a level that allows the concessionaire to earn at least an expected market return considering the risk profile of the project or a similar market asset. The cap level is also applied over the full CAPEX as the equity holder also needs to cover the debt service. An additional percent may be granted as a reward if the concessionaire maintains a predetermined service level quality.

For ease of exposition, we will describe our model in the context of an infrastructure transportation project, although it could be applied to other infrastructure projects that must be planned to accommodate growing service demand over time.

3.1. Revenue Model

We assume that a government or granting authority launches a competitive public auction to contract a concessionaire to build and operate a project monopolistically, where the tariff and guarantees are set ex-ante. We also assume that the project will be operated in a market where the revenue follows the generic function shown in Eq. (1):

$$R(T_t) = \theta \cdot T_t \tag{1}$$

where θ is the toll rate and $T = \{T_t,\ t \geq 0\}$ is the demand, i.e., the traffic level at time t.

The project's value depends on a single source of uncertainty, which corresponds to the traffic level T, exogenously defined. As is standard in the literature (Marques et al., 2022), we assume that the multiplicative shock T is the solution of the Geometric Brownian diffusion process shown in Eq. (2):

$$dT = \alpha T dt + \sigma T dz \tag{2}$$

where α and σ denote the risk-neutral drift and the volatility, respectively, $dz = \varepsilon\sqrt{dt}$, where $\varepsilon \approx N(0,1)$ is the standard Wiener process increment, and $\alpha = r + \lambda$ where r is the risk-free rate and λ is the risk premium. We assume there is an absorbing barrier, or traffic capacity limit ($s = sup$) T^S, $0 \leq T \leq T^S$) on the maximum number of vehicles that the transportation asset can accommodate within a given time frame, i.e.,

$$T_t = \min(T_t, T^S) \tag{3}$$

We also assume that the cash flow Π in year t (Π_t) is a function φ of the revenues $R(T_t)$ as shown in Eq. (4):

$$\prod_t = \varphi R(T_t) \tag{4}$$

As the cash flows generated by the project are a direct function of T_t, these will also be limited to an upper level $\Pi_t = f(T_t \mid T_t \leq T^S)$. The value of the project for a firm that invests at time $t = \tau$ is (Eq. (5)).

$$V_\tau = \int_{t=\tau}^{\tau+n} -I_\tau + e^{-\alpha t}\Pi_t \, dt \Big| T \leq T^S \tag{5}$$

where τ is the time of the investment, n is the duration of the cap and floor regime, N is the length of the contract, or the concession term, where $n \leq N$, α is the risk adjusted discount rate, I_τ is the capital investment in time τ and Π_t are the project cash flows in year t.

3.2. Cap and floor Regime Model

We assume that in addition to the concession grant, the government offers a cap and floor mechanism that specifies certain contingent limitations on the traffic revenue $R(T_t)$. These restrictions have option-like characteristics that make the cap and floor model equivalent to a collar option, which combines a Call and a Put option.

The put option guarantees the concessionaire's minimum revenue, or floor, while the call option restricts the concessionaire from earning excess revenues above the cap. The contract stipulates that the revenues will be allocated between the concessionaire and the government at discrete periods t (t = 1, 2, …, n), where $n \leq N$ is the duration of the cap and floor regime and N is the total term of the concession contract. The upper revenue threshold above which the government will receive the excess revenue (cap) is represented by R_C, and the lower threshold below which the concessionaire will be reimbursed is R_F. Both the higher and lower thresholds and exercise dates are defined as shown prior to the signing of the contract.

When $R(T_i)$ is greater than the maximum revenue threshold R_C, the concessionaire will earn an excess revenue of $\Delta_C = R(T_i) - R_C$, , $\Delta_C \in \left(0, R(T_t \mid T_t \leq T^S)\right)$ which the government has a right to receive from the concessionaire. From an option perspective, the excess sharing is a European call option with a strike price of R_C held by the government. Whenever the actual revenue at time t, $R(T_t)$ falls below the lower threshold $R_{F(t)}$, the concessionaire will receive the shortfall $\Delta_F = R_F - R(T_t)$, $\Delta_F \in \left(0, R(T_t \mid T_t \leq T^S)\right)$. From an option pricing perspective, the minimum revenue guarantee is a European put option with a strike price of $R_{F(t)}$ held by the concessionaire.

The cap and floor regime payoff $\varphi(T_t)$ for the concessionaire which combines both the MRG and the ERS at time t will then be:

$$\varphi(T_i) = \begin{cases} \Delta_C, & \text{for } R(T_t) > R_C \\ 0, & \text{for } R_{F(t)} \leq R(T_t) \leq R_C \\ \Delta_F, & \text{for } R(T_t) < R_F \end{cases} \tag{6}$$

The shortfall Δ_F received by the concessionaire if revenues fall below the floor threshold R_F is then given by $\Delta_F = \max\left(R_F - R(T_t), 0\right)$, while the payments to the government when the revenues are above the cap threshold R_C are given by $\Delta_C = \max\left(R(T_t) - R_C, 0\right)$.

Given that these are options, option pricing methods such as the real options approach must be used for purposes of valuation. Thus, we adopt the risk neutral measure to price the impact of the cap and floor regime, where the cash flows from the risk neutral traffic revenues are discounted at the risk-free rate, as is standard in the literature on option pricing. Since roadway traffic is not a traded asset, its risk premium cannot be directly observed in the market and must be determined through indirect means. Taking advantage of the fact that the value obtained by discounting the true cash flows at the project's risk-adjusted cost of capital must be the same as its value under the risk neutral measure, Freitas & Brandão (2010) showed that the risk premium λ is the solution to Eq. (7)

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$$\int_{t=\tau}^{N} E\left[\varphi R(T_t)\right] e^{-\mu t} dt = \int_{t=\tau}^{N} E\left[\varphi R(T_t^*)\right] e^{-rt} dt \tag{7}$$

where the $dT = \alpha T dt + \sigma T dz$ and $dT^* = (\alpha - \lambda) T^* dt + \sigma T^* dz$ are respectively the true and the risk neutral process for the traffic demand, and λ is the risk premium of the passenger traffic. Thus, the discounted value of the project for the cap and floor regime under the risk neutral measure at time $t = \tau,\ \tau \in [0, N]$

$$V_\tau = \int_{t=\tau}^{N} -I_\tau + e^{-rt} \omega R(T_t^*)\, dt + \int_{t=\tau}^{n} \max\left(R_F - R(T_t^*), 0\right) e^{-rt} dt - \int_{t=\tau}^{n} \max\left(R(T_t^*) - R_C, 0\right) e^{-rt} dt \quad (8)$$

where T_t^* is the risk neutral traffic demand, N is the total concession term, r is the risk-free rate, and $n \le N$ is the length of the cap and floor regime.

4. A Numerical Example

To illustrate the calculation of the cap and floor, we provide a simplified numerical example. We assume that the CAPEX is \$100 million and that the project has a life of 5 years. Annual costs include depreciation of \$20 million, and operating expenses of \$5 million, while the income tax is set at 30%. The capital structure is composed of 60% debt and 40% equity. The cost of equity (K_e) is 14%, and the cost of debt (K_T) is 6%, resulting in a Weighted Average Cost of Capital (WACC) of 8.12%. Revenues for the first year are \$31 million and although uncertain, are expected to grow at a rate of 5% each year. The project cash flows of the deterministic base case are shown in Table 1.

Table 1: Cash Flow for the base case

US$ Millions

Year	0	1	2	3	4	5
Growth rate			6.0%	-0.2%	6.2%	13.1%
(+) Revenue		**31.00**	**32.86**	**32.78**	**34.81**	**39.36**
(-) Opex		-5.00	-5.00	-5.00	-5.00	-5.00
(=) Ebtida		**26.00**	**27.86**	**27.78**	**29.81**	**34.36**
(-) Depreciation		-20.00	-20.00	-20.00	-20.00	-20.00
(=) Ebit		**6.00**	**7.86**	**7.78**	**9.81**	**14.36**
(-) Income Tax	30.0%	-1.80	-2.36	-2.33	-2.94	-4.31
(=) Nopat		**4.20**	**5.50**	**5.44**	**6.87**	**10.05**
(+) Depreciation		20.00	20.00	20.00	20.00	20.00
(-) CAPEX	-100.00					
(=) Cash Flow	**-100.00**	**24.20**	**25.50**	**25.44**	**26.87**	**30.05**

WACC = 8.12%
NPV = 4.33
IRR = 9.70%

4.1. Floor calculation

As mentioned previously, we propose that the floor for an infrastructure project should provide sufficient revenues to cover debt service obligations, maintenance and operating expenses and incurred taxes. We calculate the revenue requirements for each of these expenses separately, and sum them to calculate the floor.

We first determine an annuity payment over the five-year duration of this cap and floor regime that provides an annual cash flow whose net present value discounted at the cost of debt is equal to the $100 million CAPEX. The result is $23.74 million and an annual cash flow of that amount would allow the concessionaire to recover its initial capital expenses, but must also pay income taxes and annual operating expenses. We assume that the annual operating expenses of -$5 million are not affected by changes in project revenues, although that could be included in the analysis as well.

The annual tax payments can be determined considering a lower initial project revenue that makes the IRR equal to the cost of debt, since revenues will be low if the floor is reached. The annual required tax payments that result from this analysis are presented in Table 2. Notice that the annual tax payments increase each year as a result of the forecasted growth rate in revenues of 5 percent.

Table 2: Determining Tax allowance for the floor

US$ Million

Year	0	1	2	3	4	5
Growth			5.0%	5.0%	5.0%	5.0%
(+) Revenue		**27.61**	**28.99**	**30.44**	**31.97**	**33.56**
(-) Opex		-5.00	-5.00	-5.00	-5.00	-5.00
(=) Ebtida		**22.61**	**23.99**	**25.44**	**26.97**	**28.56**
(-) Depreciation		-20.00	-20.00	-20.00	-20.00	-20.00
(=) Ebit		**2.61**	**3.99**	**5.44**	**6.97**	**8.56**
(-) Income Tax	30.0%	-0.78	-1.20	-1.63	-2.09	-2.57
(=) Nopat		**1.83**	**2.80**	**3.81**	**4.88**	**5.99**
(+) Depreciation		20.00	20.00	20.00	20.00	20.00
(-) CAPEX	-100.00					
(=) Cash Flow	**-100.00**	**21.83**	**22.80**	**23.81**	**24.88**	**25.99**

KT =	**6.00%**	
NPV =	**0.00**	
IRR =	**6.00%**	
Revenue (t=0) =	0.000	The lowest revenue that makes NPV=0

Finally, the floor level for each year of the project is determined by adding the allowances, as illustrated in Table 3.

Table 3: Yearly floor

Allowances	1	2	3	4	5
Floor Annuity	23.74	23.74	23.74	23.74	23.74
Opex	5.00	5.00	5.00	5.00	5.00
Income Tax	0.78	1.20	1.63	2.09	2.57
(=) Yearly Floor	29.52	29.94	30.37	30.83	31.31

4.2. Cap calculation

The cap threshold is determined in a similar way. First, the annuity payments owed considering the full CAPEX value over five years at the cost of equity is computed. This provides a value of $29.13 million. The tax amount for the cap is calculated by determining the initial revenue level that results in an IRR equivalent to the cost of equity, assuming higher revenue levels near the cap threshold. The results of this analysis are shown in Table 4.

Table 4: Determining Tax allowance for cap

US$ Million

Year	0	1	2	3	4	5
Growth			5.0%	5.0%	5.0%	5.0%
(+) Revenue		**34.86**	**36.61**	**38.44**	**40.36**	**42.38**
(-) Opex		-5.00	-5.00	-5.00	-5.00	-5.00
(=) Ebtida		**29.86**	**31.61**	**33.44**	**35.36**	**37.38**
(-) Depreciation		-20.00	-20.00	-20.00	-20.00	-20.00
(=) Ebit		**9.86**	**11.61**	**13.44**	**15.36**	**17.38**
(-) Income Tax	30.0%	-2.96	-3.48	-4.03	-4.61	-5.21
(=) Nopat		**6.90**	**8.12**	**9.41**	**10.75**	**12.16**
(+) Depreciation		20.00	20.00	20.00	20.00	20.00
(-) CAPEX	-100.00					
(=) Cash Flow	**-100.00**	**26.90**	**28.12**	**29.41**	**30.75**	**32.16**

Ke =	**14.00%**	
NPV =	**0.00**	
IRR =	**14.00%**	
Revenue (t=0) =	0.000	The lowest revenue that makes NPV=0

Finally, by adding the cap allowances, we can determine the cap for each period, as illustrated in Table 5.

Table 5: Yearly cap

Allowances	1	2	3	4	5
Cap Annuity	29.13	29.13	29.13	29.13	29.13
Opex	5.00	5.00	5.00	5.00	5.00
Inc Tax	2.96	3.48	4.03	4.61	5.21
(=) Yearly Cap	37.09	37.61	38.16	38.74	39.34

4.3. Cap and floor Threshold

Following the aforementioned steps, the final values for the annual cap and floor thresholds are shown in Table 6. These values are used to limit project revenues over time, as shown also in Figure 1 and Figure 2.

Table 6: Revenue value including option

Year	Revenue	Cap	Floor	Revenue with Collar
1	$R(T_1)$	37.09	29.52	Min (39.06, Max($R(T_1)$, 30.52))
2	$R(T_2)$	37.61	29.94	Min (40.44, Max($R(T_2)$, 31.94))
3	$R(T_3)$	38.16	30.37	Min (41.89, Max($R(T_3)$, 33.37))
4	$R(T_4)$	38.74	30.83	Min (43.42, Max($R(T_4)$, 34.83))
5	$R(T_5)$	39.34	31.31	Min (44.97, Max($R(T_5)$, 36.31))

Figure 1 illustrates the base case expected revenue levels, the cap and floor thresholds and a single sample stochastic revenue path.

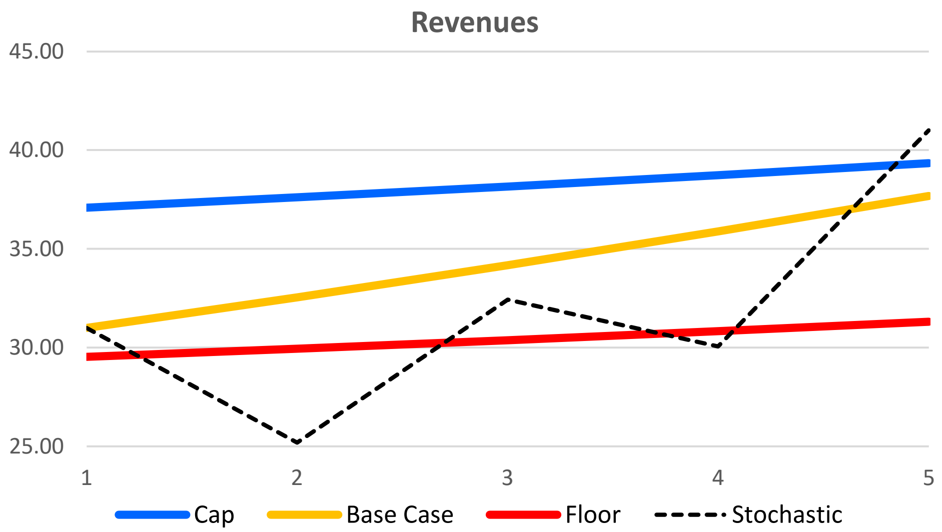

Figure 1: Cap and floor thresholds

Figure 2 illustrates the protection that the cap and floor regime provide to the private investor. On the left, several simulated stochastic revenue paths are shown. On the right, the cap and floor act as absorbing barriers for very low and very high revenue levels, reducing the overall risk of the project.

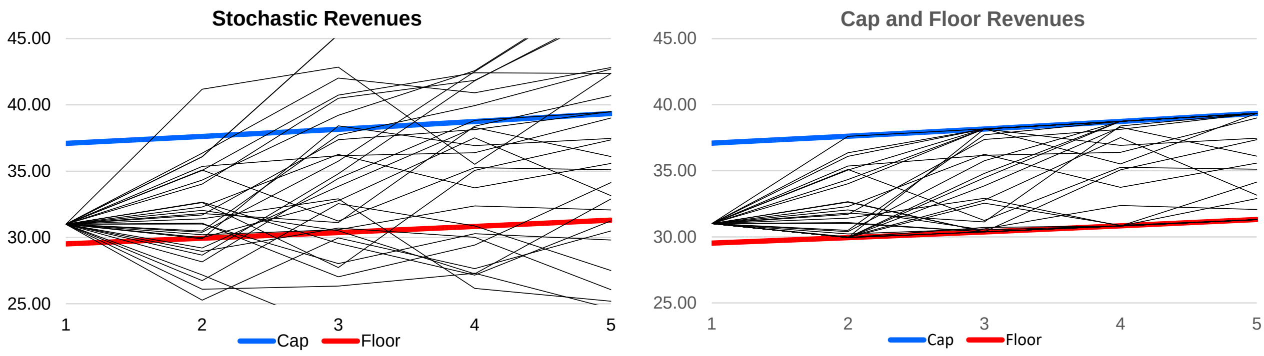

Figure 2: Project Revenues without (left) and with cap and floor (right)

5. Case Application

We develop a case application based on the BR-163 roadway concession problem described in Brandão & Saraiva (2008), which is a 1,000 mile long roadway linking the Brazilian Midwest to the Amazon river. The concession term is 25 years, the capital investment is 1.15 billion dollars, the expected debt ratio is 60%, and the cost of equity and debt capital are respectively 12% and 7% per year. The yearly risk-free rate is assumed to be 5%. Table 7 shows the basic parameters of the project.

Table 7: Base case parameters for the numerical example

Initial traffic	129,644 vehicles	r_f	5%
Concession term	25 years	K_e	12%
Revenue tax	14.03%	K_T	7%
Income tax	34%	K_d	4.62%
CAPEX	$1.15 billion USD	WACC	7.57%
Project NPV	$183.6 million USD	Debt rate	60%

The annual revenues from operating the project fluctuate following the variations of the annual traffic volume, which is assumed to be the main source of uncertainty of the project. The traffic volume simulation is obtained from the discretization of Eq.(2), and is shown in Eq. (9).

$$T_{t+\Delta t} = T_t \, e^{\left(\alpha_t - \frac{\sigma^2}{2}\right)\Delta t + \sigma\varepsilon\sqrt{\Delta t}} \tag{9}$$

where Δ_t is the time interval of the GBM simulation, which is one year for this research. α_t is the instantaneous drift rate of traffic during the time interval Δ_t, ε is a random variable that follows a standard normal distribution and σ is the annual volatility of the traffic. The volatility parameter is σ = 7%, and using the discretization of Eq. (8), the probability distribution of the project NPV not including the cap and floor options can be determined, as illustrated in Figure 3.

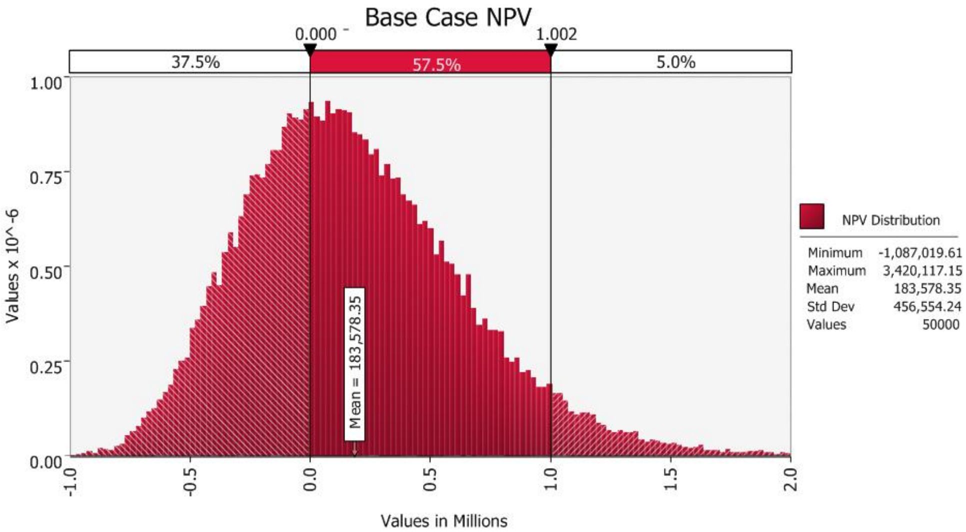

Figure 3: Probability Distribution of the NPV

The probability distribution shows that this project has a mean value of \$183.6 million, a Value at Risk (*VaR)* of a negative (\$473.8) million, a *CVaR* of (\$591.8) million, and a 37.5% chance that the NPV is negative, which indicates the necessity of providing risk mitigation clauses in the concession contract. The *VaR* metric indicates that the project has a 5% probability of losing a value greater than the stated *VaR*. The *CVaR* is the expected loss if this scenario occurs.

5.1. Setting the cap and floor Thresholds

To determine the floor allowance, we first compute the annuity owed to the debt and equity holders, considering a total capital investment (CAPEX) of 1.15 billion dollars and a cost of debt capital of 7% per year. This annuity is the minimum amount necessary for the concessionaire to honor its debt obligations covering interest and principal repayment, assuming the project is fully debt financed. Computed over the 25-year life of the project, this results in an annuity value of \$146,625 dollars. Next, the operating (OPEX) and maintenance costs in each year are added.

The tax allowance is a function of the uncertain project revenues and profits. We consider the tax that would accrue to the concessionaire in each year if the revenues were at the floor threshold, which we determine by proportionately adjusting down the expected revenue stream until the project NPV, discounted at the cost of debt, is zero. The sum of the annuity, OPEX, maintenance costs and the tax costs are then added to determine the yearly floor allowance. This result can be checked for consistency by replacing the expected revenues by the floor allowance over the life of the project, and then discounting the cash flows at the cost of debt. The result should provide a zero NPV. It should be noted that while the proportional revenue streams used to determine the floor tax allowances also provide a zero NPV, they do not adequately consider the yearly variations in OPEX and maintenance allowances. The cap allowance is determined in a similar way, considering now the cost of equity capital to determine the annuity over 100% of the CAPEX, as explained in Section 3, and adding the OPEX, maintenance and tax costs. The tax allowance is calculated in the same way as for the floor, but now using the cost of equity as the discount rate to determine the cap threshold levels that provide a zero NPV.

It is important to highlight that although the cap and floor thresholds are expected to increase annually due to the anticipated rise in the tax allowance, as a cost-based model their primary value driver is the project's capital expenditure (CAPEX), which remains fixed at $1.15 billion. This implies that the cap and floor thresholds will increase at a lower rate than the revenues, which can be detrimental to the concessionaire in the later years due to a proportionately lower floor and cap relative to the revenues. This issue can be resolved by setting the duration of the cap and floor regime to be shorter than the project's lifespan.

5.2. Real Option Valuation of the cap and floor Regime

We price the option values provided by the cap and floor regime under the risk neutral measure, where the cash flows from the risk neutral traffic and revenues are discounted at the risk-free rate. As managerial flexibilities such as the cap and floor options alter the risk of the asset under analysis, their valuation requires the use of risk-neutral pricing. This can be determined by deducting the risk premium from the asset's rate of return and then discounting the cash flows at the risk-free rate. The risk premium is the additional return over the risk-free rate that an investor requires to bear the risk of investing in a risky asset.

Given that roadway traffic is not a market asset, its risk premium cannot be directly observed in the market and must be determined through indirect means. Taking advantage of the fact that the value obtained by discounting the true cash flows at the project´s risk-adjusted cost of capital must be the same as its value under the risk neutral measure, Freitas & Brandão (2010) showed that the risk premium λ is the solution to Eq. (10)

$$\int_{t=\tau}^{N} E\left[\varphi R(T_t)\right] e^{-\mu t} dt = \int_{t=\tau}^{N} E\left[\varphi R(T_t^*)\right] e^{-rt} dt \tag{10}$$

where the $dT = \alpha T dt + \sigma T dz$ and $dT^* = (\alpha - \lambda) T^* dt + \sigma T^* dz$ are respectively the true and the risk neutral process for the traffic demand, and λ is the risk premium of the passenger traffic.

For this project the risk premium was determined to be 1.56%. A simulation of the stochastic cash flows considering the cap and floor options under the risk-neutral measure was performed. Assuming the cap and floor regime remains in place for the 25-year duration of the project, the mean of the NPV is $422.5 million. This value is significantly higher than the original value of $183.6 million of the project and is due to the compensation provided by the cap and floor regime throughout the life of the project.

The purpose of the adoption of a cap and floor regime is to make the project more attractive to the private investor by reducing its risk. As shown in Figure 3, the risk of earning a negative NPV in this project without a cap and floor is 37.5%. While the risk neutral measure provides the correct option value, it does not provide the correct probability distribution as it uses risk-neutral probabilities rather than the true probabilities. To determine the true probability distribution of the NPV, the true stochastic process of the revenues must be used. But given that the existence of the cap and floor options impact and change the risk of the project, the WACC cannot be used as a discount rate anymore as it no longer reflects the true risk of the project.

This issue can be solved by approximation. Since the NPV of the project with options computed under the risk neutral measure is known, the discount rate that provides this same NPV when modeling the revenues under their true stochastic process with the cap and floor options can be determined. After several simulation runs, we determined that the discount rate that provides an NPV close to $424.5 million is 5.79%. The true probability distribution considering this discount rate is shown in Figure 4. Both the upper and lower tails of the distribution have been truncated due to the effects of the cap and floor mechanisms which effectively eliminate excessively high and low values, respectively.

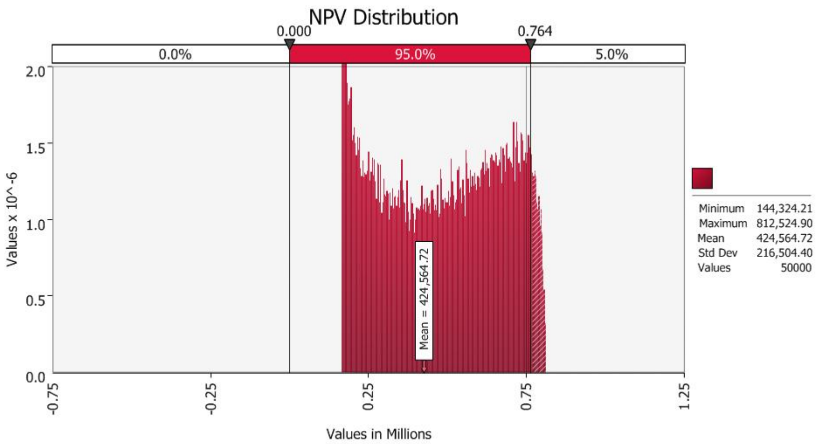

Figure 4: True probability distribution using the true risk adjusted discount rate of 5.79%

Under a full-term cap and floor regime, the project risk is substantially reduced and the probability of the project earning a negative NPV becomes zero. Likewise, both the *VaR* and the *CVaR* are also zero, as this mitigation mechanism reduces the risk of the project to the point that even for the worst 5% scenario the expected NPV is still positive. The appropriate discount rate for this risk is reduced to 5.79%, which reflects the lower risk of the project under the cap and floor regime.

5.3. Cap and floor Regime terms

Unlike electricity concession projects, in the transportation sector it is common to have risk mitigating mechanisms that are in force for less than the full term of the concession (Marques et al., 2024; Sant´Anna et al., 2022). Figure 5 shows the true probability distribution of the project NPV for different time lengths which were computed as explained for Figure 4, with different discount rates for each one. We can see that as the cap and floor regime is shortened, the risk to the concessionaire increases.

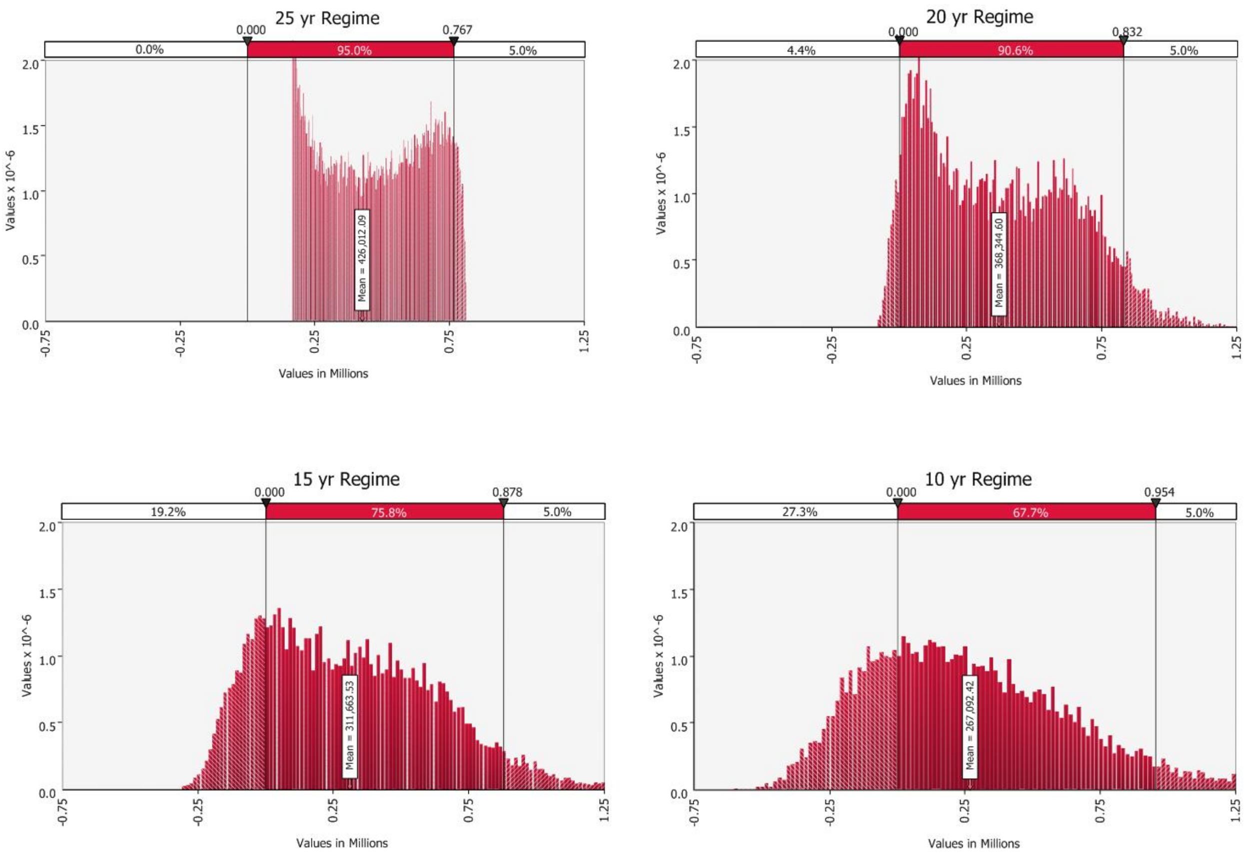

Figure 5: True NPV probability distribution under different cap and floor regimes

While the full 25-year cap and floor regime eliminates all the downside risk for the concessionaire, this changes with the length of the term, as can be seen in Table 8. The base case and the 5 and 10-year regimes indicate a high level of risk with a 5% probability of having losses equal to 3.2, 2.5 and 1.9 times the expected base case NPV of 183.5 million, respectively. On the other hand, for the 20 and 25-year regimes, the *VaR* is positive, and the *CVaR* and P(*NPV*<0) indicate minimal risk, suggesting that these regimes provide an almost risk-free project for the concessionaire.

Table 8: Risk Metrics for different cap and floor regimes periods

Regime	None	5 yrs	10 yrs	15 yrs	20 yrs	25 yrs
VaR	(473,811)	(370,781)	(295,034)	(132,065)	0	0
CVaR	(591,828)	(468,389)	(351,264)	(178,982)	(20,992)	0
P(NPV<0)	37.50%	32.90%	27.30%	19.20%	4.40%	0.00%
NPV	183,578	230,911	267,092	311,663	368,344	426,012
Expected Cost to Gov	0	47,333	83,514	128,085	184,766	242,434

Figure 6 shows the effect of a 15-year cap and floor regime on the NPV probability distribution of the project. The distribution in red is the original base case NPV distribution, where there is no risk mitigation, while the distribution in blue represents the 15-year regime true NPV distribution. The tails of the 15-year distribution are shorter due to the effect of the cap and floor, resulting in a reduction in the risk as noted by the lower standard deviation.

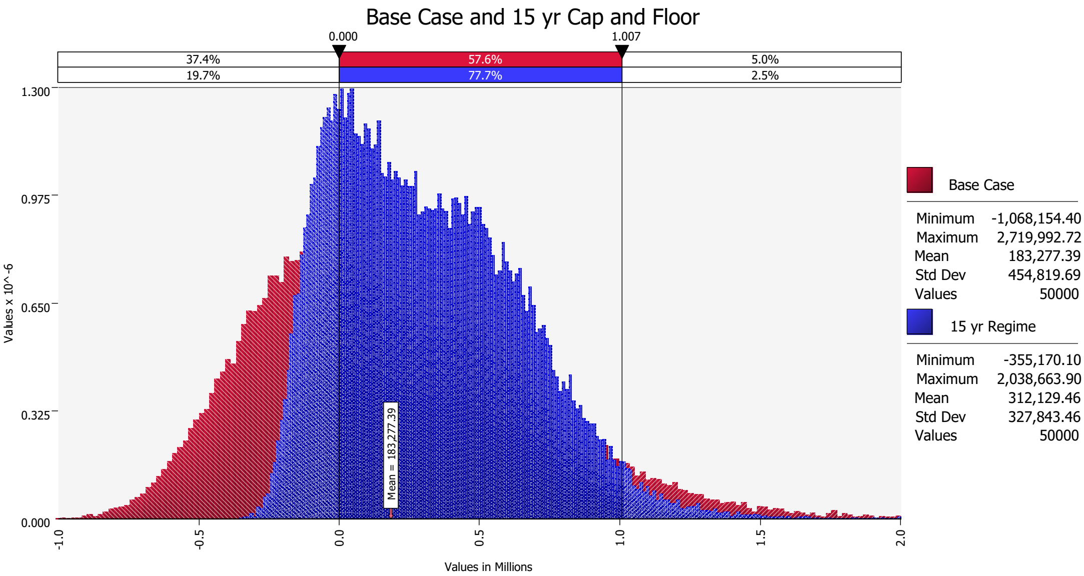

Figure 6: NPV True Probability Distribution for the Base Case and 15-year Regime

6. Conclusions

The shift towards private capital funding for infrastructure projects in transportation in the past few decades requires that the government provide effective risk mitigation mechanisms to attract investors for projects where demand risk is deemed to be excessive. Nonetheless, these mechanisms represent contingent liabilities for the government which may impose significant costs on taxpayers if they are not adequately evaluated and priced. Unfortunately, there are no clear rules and procedures that can guide governments to optimally set these guarantee levels, which frequently result in negative outcomes and overly burden the taxpayers.

In this article we developed a cost-based mechanism where the main advantage of this method is that it offers a simple, clear and effective guideline to the government on how to create the risk mitigation regime. While cap and floor, or collar options, have been widely used for concession projects in this area, threshold levels have been arbitrarily set historically as a percentage of revenues, rather than a function of costs as we propose here. To the best of our knowledge, this is a novel contribution to public policy in transportation infrastructure projects.

The proposed model offers significant implications for real-world applications, particularly in the context of PPP infrastructure auctions. This model provides a structured, clear, and logical approach to setting threshold levels, creating a balanced framework for both private investors and government agents. The model reduces perceived uncertainties for private investors, fostering greater confidence and encouraging participation in PPP projects. This, in turn, enhances competition, leading to more innovative and cost-effective solutions for infrastructure development.

Our results show that fair levels of floor for the MRG and cap for the ERS can be determined that minimize the cost to the government while assuring a reasonable level of support for the concessionaire to undertake the project. The cap and floor regime offers benefits for all stakeholders due to its simplicity and transparency. For government and policymakers, it provides a clear guideline for setting the threshold levels, that ensures an expected return sufficient to encourage participation by the private sector. The model enhances cost efficiency by minimizing the financial burden on public funds and taxpayers, and the use of a cost-based methodology to determine thresholds ensures that payouts remain aligned with project realities, avoiding unnecessary financial liabilities from revenue shortfalls or windfall profits. This mechanism not only protects public resources but also creates a predictable and transparent framework for negotiations and contract formulation, promoting trust among stakeholders and facilitating smoother project implementation. For private investors and concessionaires, the model's structured approach guarantees that, at a minimum, all their third-party obligations will be met, contributing to the stability and success of PPP projects and mitigating bankruptcy risk. This proper risk allocation is crucial for maintaining the financial health of the projects and for attracting long-term investments from private entities.

While the model draws inspiration from its application in electricity markets, its adaptability extends to various infrastructure sectors, including transportation, energy, and sanitation. Its flexibility allows it to accommodate the unique revenue dynamics and risk profiles of diverse projects, making it a versatile tool for future PPP auctions. Furthermore, the structured approach provided by the cap and floor model offers valuable insights for shaping policy decisions and regulatory frameworks, ultimately leading to more efficient and sustainable infrastructure development

The broader adoption of this model can contribute to more efficient and attractive PPP auctions, with potential to inform future research on its long-term effects. Future studies could explore the model's impact on project performance, investor behavior, and public sector financial health, offering critical data for continuous improvement. These real-world implications highlight the model's capacity to drive innovation and enhance the success of PPP infrastructure projects.

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