

# A Game Theoretical Approach for Seasonalization of Hydropower Plants Physical Guarantee

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## Overview

In 2022, Brazil harnessed an impressive 440.0 terawatt-hours (TWh) of hydraulic energy, constituting a substantial 61.9% share within the total national electric matrix, which stood at 690.1 TWh (EPE, 2023). Hydropower, being a renewable energy source, holds a crucial position in the country's energy landscape. Nonetheless, this dominance also brings forth notable complexities in the field of system management, mainly due to the dynamic nature of river flows and reservoir levels.

Within this intricate context, the effective management of physical guarantees and their seasonalization emerges as a critical facet in the orchestration of the Brazilian energy system. The concept of physical guarantee represents a cornerstone in the management of hydroelectric power generation within the Brazilian Interconnected System. It embodies the maximum energy commitment a power plant can make in its contracts over a one-year period. Although the total physical guarantee cannot change for the year, there is flexibility for the individual plants to allocate and distribute their annual physical guarantees on a monthly basis, with the total allocation remaining constant. This practice allows power plants to better align with their energy supply contracts and risk profiles, a process referred to as the seasonalization of the physical guarantee.

This work presents a novel approach employing game theory principles and time series forecasting models to optimize the allocation decisions concerning the seasonalization of the physical guarantee. Our approach incorporates insights from regulatory agencies, market forecasts, and industry expertise.

It is important to note that the seasonalization of the physical guarantee is an annual process with significant financial implications for the market participants, especially for the hydroelectric power plants. This process directly influences the exposure of each stakeholder to the weekly spot price (PLD) that is contingent on factors like anticipated rainfall patterns and reservoir levels. For instance, the most risk-averse strategy involves strictly aligning the yearly physical guarantee with the monthly requirements of energy supply contracts, effectively mitigating individual exposure to the PLD. Conversely, agents with a higher risk appetite may opt for a more dynamic approach, allocating more physical guarantee to months where spot prices are expected to peak and accepting increased exposure to price fluctuations.

The proposed approach empowers decision-makers to calculate the monthly physical guarantee for their power plants, which considers the other agent's decisions. Notably, we show that regardless of other agent's risk preferences, the optimal decision consistently converges to the same strategy, known as the Nash Equilibrium, optimizing overall results.

## Methods

Game theory has found a multitude of applications within the electricity sector, serving as a crucial tool for tasks such as determining energy prices within specific markets and establishing the values in various auction scenarios. An innovative approach in this domain has been presented by Abapour et al. (2020), who leveraged game theory to devise a strategy for Demand Response (DR) aggregators. This strategy aims to maximize profit by harnessing the flexibility of customers' appliances to curtail energy costs, while also employing game theory to model competition between aggregators.

The process of monthly seasonalization, conducted once a year, can be formulated as a strategic game. Every participant's decision during the seasonalization process has a direct impact on other participants results, through the GSF (Generation Scaling Factor), which is the adjustment factor of the generation, above or below the physical guarantee. For instance, if one agent maximizes their power plant output for a particular month based on a bullish forecast for the PLD, and all other agents do the same movement, a mismatch between generated energy and the allocated physical guarantee may occur. This imbalance could lead to the GSF dropping below 1, as the physical guarantee will probably surpasses the energy generated for that month. When the GSF falls below 1, it results in financial penalties for all participants.

Therefore, successful seasonalization strategies must consider the price forecast and the decisions of other agents, which can significantly influence the GSF and impact the power plant revenue. This complex situation can be modeled as a strategic game. For the analysis of the dynamics involving the physical guarantee seasonalization, we will consider the perfect competition environment, where the participants are free to enter and leave the market and there is no possibility of collaboration or collusion between the agents (non-cooperative game). This assumption is reasonable given that the market for hydro power generation is well established and has many participants.

New participants can freely enter the market through the Regulated Contracting Environment or the Free Contracting Environment. For non-cooperative games, the most important concept developed is Nash Equilibrium, which represents the set of strategies in which each player cannot individually get their individual result (payoff) better by changing the strategy, unilaterally. This equilibrium can be deterministic or probabilistic.

When it comes to the nature of information within the seasonalization game, we classify it as a game of imperfect information. This

categorization stems from the fact that each agent's seasonalization decisions are revealed simultaneously, with no opportunity for subsequent revisions. Information is deemed perfect when all participants are fully aware of the actions taken by their counterparts. In the context of seasonalization, however, agents only become privy to the decisions made by other participants after they themselves have committed to their choices, given that these decisions occur concurrently.

While it is true that there are several pieces of information held by individual agents that remain inaccessible to others, the critical information required for informed decision-making during the seasonalization process is openly available. This includes the set of potential strategies, making this game fall under the category of complete information games.

Before delving into the intricacies of the seasonalization game, it is essential to establish two fundamental concepts: the GSF and the payoff for each agent. The GSF serves as a crucial index, gauging the extent of coverage the system offers for the contracted energy. It can be precisely defined using equation (1).

$$GSF_j = \frac{\sum_{i=1}^n \tau_{ij}}{\sum_{i=1}^n g_{ij}} \quad (1)$$

where  $\sum_{j=1}^{12} g_{ij} = G_i$ ,  $G_i$  is the annual physical guarantee of power plant  $i$ ,  $GSF_j$  is the GSF for the month  $j$ ,  $\tau_{ij}$  is the total generation of power plant  $i$  for the month  $j$ ,  $g_{ij}$  is the physical guarantee allocated by power plant  $i$  for the month  $j$  (this value is obtained from the seasonalization of the physical guarantee) and  $n$  is the number of power plants in the energy relocation mechanism.

The annual payoff for each agent ( $\pi$ ), as a result from the seasonalization decision, is determined by summing the power plant's results over the course of the twelve months in a year as shown in equation (2).

$$\pi_i = \sum_{j=1}^{12} P_j \cdot [(g_{ij} \times GSF_j) - C_{ij}] \quad (2)$$

where  $P_j$  is the spot price (PLD) for the month  $j$ ,  $C_{ij}$  is the contracted amount from power plant  $i$  for the month  $j$ .

The Nash Equilibrium, when it exists, represents the set of strategies in which each player cannot individually get their individual result (payoff) better by changing the strategy, unilaterally. The analytical solution the problem must be one of the solutions in which the first derivative of the payoff function equals zero. The same applies both to player 'a' and player 'b'.

Our approach was to find the function points where the derivative of the function is zero and that are the maximum of the payoff function, what means that the second derivative of the function must be negative. Thus, we calculated the point where the derivative is equal to zero for the payoff function of one player and then we substituted the results obtained on the other player payoff function. The Nash Equilibrium is found if we can make sure that the optimal solution cannot be improved, for both players, by individually changing their position.

## Results

The chosen player for the numerical application possesses an installed capacity of 700 average Megawatts, with a corresponding physical guarantee of 500 average Megawatts. By considering the proportion of the total physical guarantee contributed by this player to the entire system, we can determine the optimal allocation of physical guarantee for the player, along with the associated monthly payoffs in BRL MM. The player total payoff for the seasonalization game is BRL -14.68 MM. This value is consistent with prediction for the system condition, where the gross generation will be substantially lower than the physical guarantee.

## Conclusions

Our model suggests an optimal strategy that simultaneously maximizes a player's results while safeguarding them from potential reductions in payoffs resulting from other participants' actions. This strategy revolves around achieving equal equivalent prices, where the equivalent price for a given month is determined by multiplying the GSF by the PLD. Remarkably, this strategy, which optimizes payoffs, is also the most conservative and is advisable even for risk-averse participants.

This study applied this optimization model to a key market player. We designed a proprietary model to predict energy demand and estimate gross energy generation, employing a Seasonal Autoregressive Integrated Moving Average (SARIMA) model with dummies. The player authorized and monitored our model's application, and the results significantly informed their seasonalization process for the year 2021.

For future research, we propose evaluating the seasonalization game formulation using utility as a payoff metric, which would be determined based on a participant's risk profile, rather than focusing solely on financial returns. Additionally, exploring the existence of a Nash equilibrium and identifying dominant strategies within this context could further enhance our understanding of this complex process. In future research, it also may be valuable to examine the equilibrium by considering energy load and PLD forecasts as stochastic variables. This approach could potentially yield novel insights distinct from those presented in this current study.

## References

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