Materials, Technology and Growth: Quantifying the Costs of Circularity

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Abstract

Environmental concerns over growing raw material extraction and waste generation have led many governments, including the United States, to introduce policies intended to reduce the extraction of new materials from the earth and boost material recycling. In the policy sphere, this is referred to as circularity. This paper develops a quantitative growth model with material use and directed technical change to quantify the costs of circularity policies. We study the United States goal of 50% recycling by 2030 and find it would require doubling recycling subsidies and cost 0.17% in consumption-equivalent welfare. However, this policy would also increase virgin extraction. Achieving a substantial reduction in new material extraction itself is very costly. Returning to 1970 levels of extraction entails a long run consumption cost of 6% and lost growth of 1.1% per decade.

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1 Introduction

In 2022, over 100 gigatonnes of material were extracted from the earth. This represents a 14-fold increase relative to material extraction in 1900, 90% of which has occurred since 1950, and 60% of which has occurred since 1980 alone (Krausmann et al., 2018, 2009). The increasing pace of material extraction is mirrored in the growth of waste generation. The most recent estimates find that just over 2 gigatonnes of waste were generated in 2016, representing a 55% increase in the six years since the last report (Hoornweg and Bhada-Tata, 2012; Kaza et al., 2018).[1] Environmental concerns regarding these patterns have led to calls from within governments, the popular press and academia to reduce extraction of new materials and increase material recycling – i.e., to improve "circularity".[2] In the policy sphere, the United States (US) Environmental Protection Agency (EPA) announced a target of a 50% recycling rate by 2030.[3] The European Union (EU) has proposed a tax on the use of virgin plastic inputs as part of its Circular Economy Action Plan (CEAP, 2020). Indeed, all 191 United Nations member states are working towards the 2030 Sustainable Development Goals which promote circularity as an explicit target (UNSDG, n.d.). This paper provides the first estimates of the macroeconomic costs of circularity policies, with a focus on the US. We find that augmenting existing subsidy policies to achieve the US recycling target is relatively inexpensive in welfare terms. However, such a policy actually increases the extraction of virgin materials in the medium to long run. A significant, long-term reduction in virgin extraction itself is very costly.

This paper makes three contributions to the literature. First, we introduce two data sources – Material Flow Accounts (MFAs), which track economy-wide material use (Eurostat, 2018) and EPA data on waste generation and recycling (EPA, 2020) – to the macroeconomic literature and show how they can be mapped to standard macroeconomic aggregates. Combining the constructed dataset of material flows in the US economy from 1970-2015 with historical material prices, our second contribution is to show that material dynamics can be understood through the lens of profit-maximizing behaviour of final goods producers and

[1] These patterns have had significant, negative impacts on our environment. Material extraction and processing are responsible for over half of global greenhouse gas (GHG) emissions and 90% of land-based biodiversity loss (UNEP, 2024). In the United States, waste landfills account for 14.3% of annual methane emissions (Cusworth et al., 2024).

[2] In a recent NBER Working Paper, Fullerton (2024) explicitly pushes economists to think more concretely about the economics of circularity (see also Fullerton et al., 2022; Stahel, 2016). Circularity is considered instrumental in the mitigation of GHG emissions (European Commission, 2012) and recycling policies are becoming increasingly prominent (Kinnaman, 2006).

[3] The EPA calculates the municipal solid waste (MSW) recycling rate as the ratio of total MSW recycled to total MSW generated (total MSW recycled plus total MSW disposed of). See EPA (1997, 2020, n.d.) for details of this statistic, components of MSW, and 2030 recycling target.

material recyclers. This is in contrast to the existing emphasis on households in the recycling literature. Third, and most importantly, we develop a quantitative, directed technical change (DTC) model of material use, recycling and growth that can be directly mapped both to national income and material flow accounts, and reproduces well the historical paths of material use and recycling. The calibrated model is used to study the future path of the recycling rate (the primary policy measure in the US), output and consumption growth under different scenarios. We consider the economic costs of alternative policies to achieve the US EPA goal of a 50% recycling rate by 2030, as well as the costs of returning to the 1970 level of virgin material extraction (a reduction on the order of 40% relative to current levels).

To provide a comprehensive characterization of material use and circularity in the US economy, we combine data from MFAs and EPA with production data from the US National Income and Product Accounts (NIPA). Although they have been largely overlooked in the literature, we believe MFAs have the potential to be a valuable source of macroeconomic data and we discuss them in detail.[4] We document that over the last forty years, the US economy has produced more output per tonne of material used, has shifted toward the use of recycled materials and is recycling an increasing share of waste materials for reuse.

Using historical price data, we then provide novel evidence that circularity itself is a market phenomenon. We document that the aggregate recycling rate co-moves strongly with material prices, and that the long term trend to reduce material use occurs in concert with a steady rise in the relative price of virgin materials. A second contribution of our paper, both empirically and conceptually, is to show that prices are crucial to understanding aggregate recycling and material dynamics.

Guided by the empirical evidence, we next build a growth model with recycling and virgin extraction sectors in which material dynamics are driven by market prices as well as government policy. We take inspiration from a wide literature studying the interactions between resource use and recycling going back to Smith (1972), growth models of DTC (Acemoglu, 2002; Acemoglu et al., 2012) and models of growth and material use such as Di Vita (2001), Akao and Managi (2007), Pittel et al. (2010), and Lafforgue and Rouge (2019).[5] In our

[4]There are few exceptions that primarily focus on material intensity (i.e. the amount of materials required per unit of GDP), e.g. Behrens et al. (2007); Krausmann et al. (2008). A large literature in industrial ecology studies and develops MFAs. See, for instance, Fischer-Kowalski et al. (2011); Giljum et al. (2014); Schandl et al. (2018). Krausmann et al. (2018) update the existing global domestic extraction series (Krausmann et al., 2009) which distinguishes around 150 materials that are aggregated to four main groups: biomass, fossil energy carriers, ores and non-metallic minerals. Similarly, the aggregate EPA data on waste generation and recycling have received scant attention in the macroeconomic literature because most empirical studies on waste and recycling leverage policy-variation for analysis.

[5]For a broader overview of existing macroeconomic modeling of resource use and the circular economy, see McCarthy et al. (2018) and citing literature. See also Fullerton (2024), Sørensen (2017).

model, profit-maximizing recyclers choose how much waste to recover. This highlights a key argument of the paper, i.e., circularity itself is a market phenomenon, responding to material prices. Our emphasis on the profit motive as a driver of aggregate recycling is in contrast to the existing literature which focuses largely on household choices, e.g., garbage pricing (Fullerton and Kinnaman, 1996), residential curbside recycling (Kinnaman, 2006), bottle return refunds (Beatty et al., 2007; Ashenmiller, 2011; Viscusi et al., 2011), and carryout bags policies (Taylor, 2020).[6] We argue, supported by the aggregate evidence, that the firm side is equally important. Household choices indeed matter – they must participate in diverting waste material to recycling – but recovery must also be profitable for recyclers.

We highlight two key features of our theory. First, we explicitly include material flows in a way that the model can be directly mapped to the data. In our model, materials *circulate* through the economy, from producers to consumers and back to producers again as recycled material input. The resulting *materials law of motion* (MLOM) that we derive governs the evolution of the mass of materials embodied in stocks of consumption and has direct empirical counterparts in the data. Second, we include a rich structure of DTC, which allows for the differential evolution of three separate total factor productivities (TFP): a capital-labor-saving technology, a material-saving technology and a recycling technology. In this structure, resource-saving innovations respond to relative input expenditures over time. Our formulation of DTC is in line with that of Hassler et al. (2021), while the essence of our framework is related to Zhou and Smulders (2021), who theoretically characterize the innovation tradeoffs of improving material recycling in the presence of endogenous technical change.

The third, and we believe primary, contribution of this paper, is quantitative – mapping the data and theory described above to obtain a calibrated model that can reproduce historical patterns of material use, recycling and growth. As a first step, we examine the accuracy of our materials accounting framework, captured in the MLOM. We test whether it can reasonably capture material flows in the US economy over the period 1980-2015 and find that it does. For calibration, we use a version of the model with exogenous virgin material prices and feed in the empirical paths of prices and recycling policy. We initialize the economy in 1980 and demonstrate that the calibrated model can reproduce well the recycling rate, material intensity, output growth and recycler's TFP growth rates for the US economy, as well as several non-targeted moments (the material mix, the expenditure share of virgin material and the capital-output ratio). This calibrated model is the basis on which we quantify the costs of achieving material use targets. Our focus on a quantitative model

[6] For theoretical studies focused on the household side, see, for instance, Palmer et al. (1997), and Fullerton and Wolverton (2000).

of growth and resource use with DTC relates our work to a recent set of papers, e.g., Fried (2018) and Casey (2023), who study the quantitative implications of policy for energy use and growth. Our analysis shares many features and the spirit of these papers but our focus on materials demands a specific production structure and material flow accounting absent in models of energy use. In line with this literature, we find an important role for endogenous, resource-saving technology responses to policies.

With the calibrated model in hand, our first exercise quantifies the historical role of policy, prices and technology in explaining the path of recycling between 1980-2015. The expansion of recycling subsidies in the 1980s made a modest contribution to the overall path in the US, contributing 4 percentage points to the recycling rate by 2015. Lower virgin material prices after 2000 also played a role – constant 1980 prices add 3 percentage points to the recycling rate in 2015 (at 38%). However, the starkest finding is the role of technology. Absent recycling technology growth, recycling would have declined to 10 percentage points by 2015. While the no-technology case is extreme, endogenous technology responses play a role even in the policy and price counterfactuals. Lower subsidies and higher prices both shift innovation towards material technologies, a dynamic that grows in importance over longer time horizons.

In our second quantitative exercise we use the model to study the costs of achieving the US target of a 50% recycling rate by 2030. Motivated by existing policy, we consider two alternatives. First, we examine a subsidy to the recycling cost, as used in the US. Achieving the US recycling target requires a doubling of the current subsidy, from covering 42% of costs in 2015 to 87% of costs in 2024. We find that this policy is not very costly in consumption-equivalent terms relative to the benchmark, implying a 0.17% decline in welfare and 0.22% increase in output by 2100. For comparison, we then consider a welfare-equivalent tax on virgin material, as imposed in the EU. This tax (83.5%) yields a maximum recycling rate of slightly above 40% in 2033. Output growth falls by 2/100ths of a percent and by 2100 this yields a 1.77% decline in output relative to the benchmark. In terms of achieving a higher recycling rate, the model is clear that a recycling subsidy, as pursued by the US, is the preferred option.

Our findings suggest that achieving the US recycling target is relatively inexpensive. Indeed, comparing cost-equivalent policies, a subsidy also delivers a greater increase in the material mix, which is the preferred policy measure in the EU. However, because the recycling rate or material mix are ratios, raising these through subsidies conceals changes in levels that seem undesirable. In our policy exercise, doubling the recycling subsidy implies levels of virgin extraction that by 2080 are above those in the benchmark economy. This happens because subsidies lower the cost of recycled materials, which our analysis shows are weak complements to virgin materials. As the overall cost of the material composite falls, total material usage rises. If the concern underpinning material use policies is to reduce stress on the natural environment through extractive activity, a subsidy policy may be pushing the economy in the wrong direction.

To put this in context, we conduct an exercise where the target is a substantial reduction in levels of material extraction. Imposing a virgin material tax that reduces virgin extraction in 2024 to the level of 1970 requires an enormous tax on virgin materials of nearly four-hundred percent (397%). This leads to an immediate reduction in virgin extraction on the order of 40%, growing to a 61% reduction (relative to the benchmark) in 2100. Incidentally, this policy achieves a 50% recycling rate by 2030 and maintains a recycling rate above the 2015 level out through 2100. However, such a reduction in virgin extraction comes at great cost. There is a 1.42% decline in consumption-equivalent welfare, which understates the long-run costs since the gap in consumption relative to the benchmark grows over time. By 2100, aggregate output is 7.22% lower under this policy and consumption lower by 5.8%. Output growth falls by 1.1%-per-decade, driven by a shift in innovation toward material technologies. While striking, these estimates are also quite robust to alternative assumptions about the future path of material prices and parameter values.

The paper continues as follows. Section 2 describes MFAs and EPA data in detail and provides evidence for the importance of prices in understanding aggregate material and recycling dynamics. Section 3 develops a material accounting framework, tests it against the data and then nests it in a growth model with DTC. And, Section 4 calibrates the model on historical data and simulates the model forward to analyze alternative policy scenarios.

2 Empirical Motivation

In this section, we describe how we combine a variety of data sources to give a picture of material use and circularity in the US from 1970 to 2015.[7] We also present the main patterns of circularity in the US and three facts that emphasize the role of economic forces in driving historical material dynamics.

2.1 Data Sources and Historical Patterns

A comprehensive characterization of material use and economic circularity in the US economy requires data on material inflows and outflows at the national level. To measure aggregate material inflows we use the United Nations (International Resource Panel) Global Material Flows Database. Material Flow Accounts (MFAs) have been developed as a satellite

[7]Appendix A.1 contains all details for producing the time series of quantities, prices and subsidies described below. We also provide links to the original sources.

of the System of National Accounts (SNA; NIPA in the US) with which economists are much more familiar.[8] Just as the SNA tracks the flow of production in an economy, the MFAs provide a framework to track the flow of materials, both domestically and across borders through international trade. MFAs separate materials into four primary categories: biomass, metal ores, non-metallic minerals and fossil energy, with each category then subdivided into more detailed layers. We use domestic material consumption as our measure of material inputs.[9]

To measure material outflows and recovery, we use waste generation and recycling data from the EPA, which provides consistent measures of these series beginning in 1960. We use the EPA's primary national measure of waste generation, constructed using a materials flow methodology. Conceptually, this means that waste generation numbers are estimates based on past consumption data, estimates of product life-cycles and material content. The recycling series computed by the EPA combines estimates of materials generation with survey data from recyclers on recycling rates for different materials collected (EPA, 2022). It is worth emphasizing that EPA data focus exclusively on Municipal Solid Waste (MSW). MSW includes all residential, commercial and institutional waste excluding industrial and construction waste.[10] These data, while incomplete, are the most comprehensive, longitudinal measures of waste and recycling in the US that we know of.

The main patterns of material use are presented in Figure 1. The recycling rate, the primary policy measure of circularity in the US, captures the recovery of materials after use. In the US, the recycling rate (share of material waste recycled) has increased by a factor of four since 1970. Two other related measures are also presented in Figure 1: the material share of output and the (recycled-virgin) material mix. The material share of output captures an economy's reliance on materials to produce goods and services while the material mix represents the relative use of recycled materials. Together, the material share of output and the material mix convey an economy's reliance on virgin materials to produce output. The total (virgin plus recycled) material share of GDP has been falling over the past 60 years,

[8]These accounts were initially constructed in piecemeal attempts for specific years and countries (see Adriaanse et al. (1997) for details.), but have since moved towards a standardized framework in the OECD (OECD, 2008). For most OECD countries data is available going back to at least 1960.

[9]Domestic material consumption is the total materials consumed in the domestic economy (sum of domestic extraction (DE) and net exports) and, in principle, it captures the net inflow of all materials into the economy, including from products produced abroad. Our measurement approach is consistent with measures produced by the European Environment Agency. Their Circular Material Use Rate (EEA, 2024) is computed as the share of recycled material to total material, where total material is the sum of recycled and domestic material consumption. It conveys the same information as our measure of the material mix (recycled-virgin materials ratio).

[10]These exclusions are important to note because, while industrial and construction waste are not easily measured, existing estimates suggest they are the vast majority of waste generation by weight (EPA, 2023; Purchase et al., 2021). See EPA (2022) for a more detailed description of methodology.

while the ratio of recycled to virgin materials has increased seven-fold. In summary, over the last forty years the U.S. economy has produced more output per tonne of material used, has shifted toward the use of recycled materials and is recycling an increasing share of waste materials for reuse.

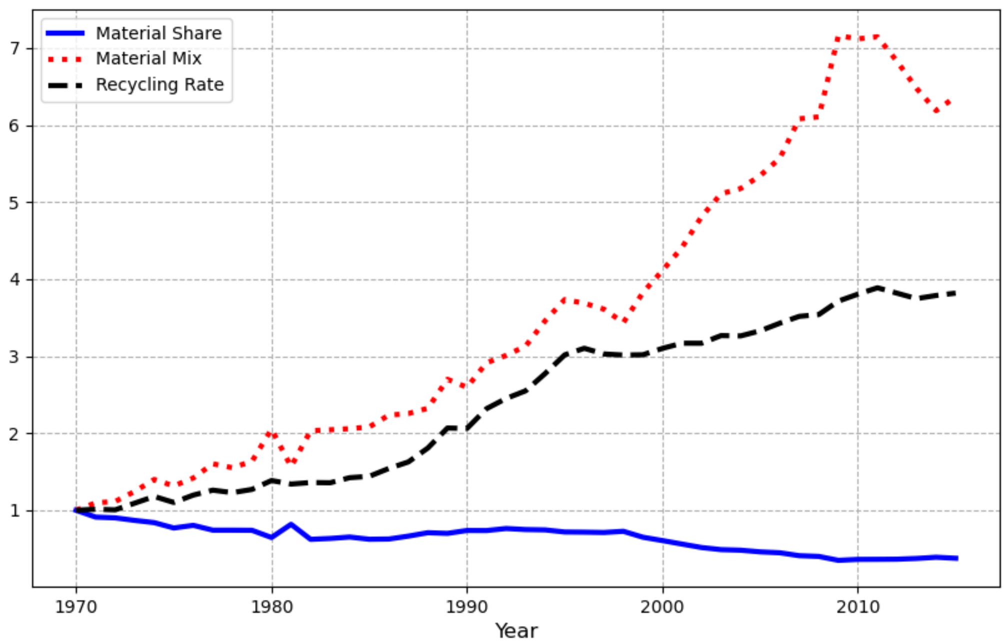

Figure 1: Material share is calculated as the quantity of total materials - virgin (TPM_EEC) plus recycled) divided by GDP. Material mix is the ratio of recycled to virgin materials. The recycling rate is calculated using information from the US EPA Advancing Sustainable Material Management 2018 Tables and Figures, December 2020. Period: 1970 - 2015. Data is normalized to 1970 values (1970 = 1). See Appendix A.1.1 for details.

2.2 The Economic Drivers of Circularity

In order to investigate the economic drivers of circularity in the US, we need to combine our data from MFAs, EPA and NIPA with historical material prices, constructed using material-specific price indices weighted by mass.[11] We document three facts. First, Figure 2 (Panel A) shows that as the relative price of virgin materials (dashed black) has risen, the material share of output (solid blue) has declined and the material mix (dotted red) has increased. These material input patterns are consistent with standard input substitution logic from a producer's problem, and suggest that the aggregate material share and material mix are strongly influenced by the virgin material's price. The model we construct in Section

[11]See Appendix A.1.1 for details.

3 will be consistent with these patterns.

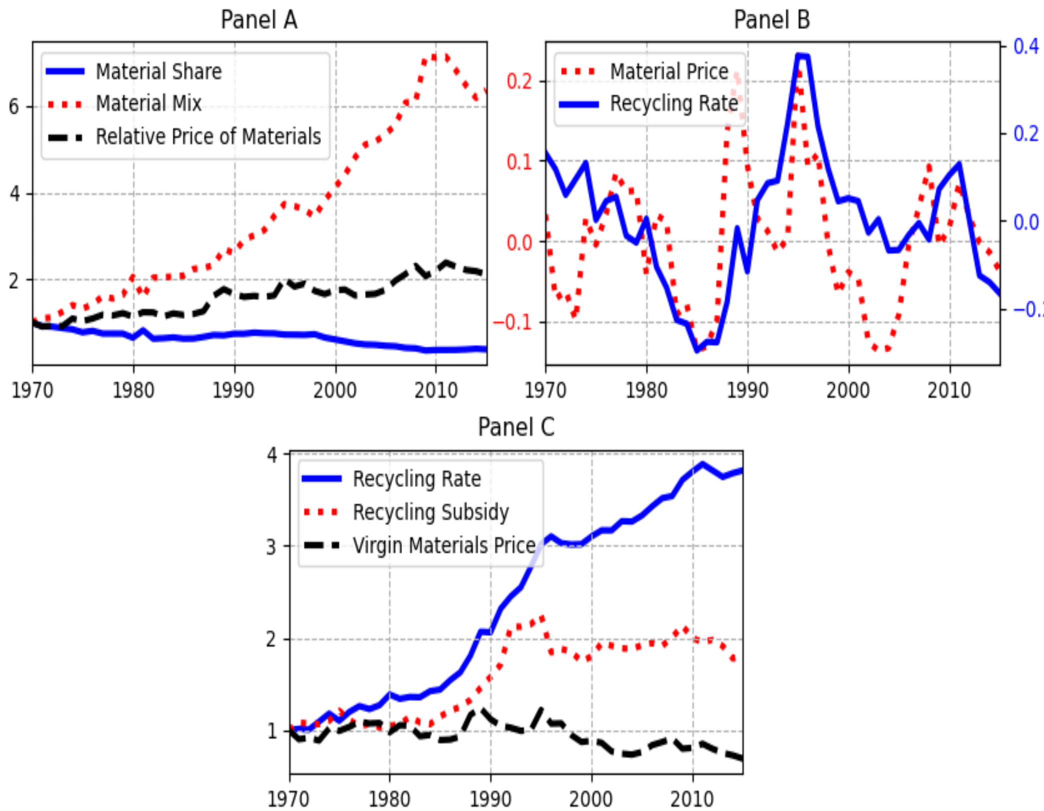

Figure 2: *Panel A*: Material Share is calculated as the quantity of total materials (TPM_EEC plus recycled) divided by GDP. Material mix is the ratio of recycled to virgin materials. Relative price of materials represent the ratio of (virgin) materials price and price of capital (Relative Price of Investment Goods - PIRIC). *Panel B*: De-trended (HP Filter) Material Price and Recycling Rate. *Panel C*: Estimates of recycling subsidies per tonne of waste (Appendix A.3.5). Data are taken from OECD.Stat, Global Material Flows Database, FRED, and US EPA Advancing Sustainable Material Management. Period: 1970 - 2015. Data is normalized to 1970 values (1970 = 1).

Second, recycling dynamics co-move positively with material prices (correlation of 0.44), plotted using the cyclical component of HP filtered price series (dotted red) and recycling rates (solid blue) in Figure 2 (Panel B). Profit maximizing recyclers recover more materials when prices rise. We view this as strong suggestive evidence that the recycling rate is driven by the profit motive of recyclers as much as anything else.

Finally, we document that policy appears to have played an important role in historical patterns of circularity. Increasing awareness of environmental issues starting in the middle of the 20th century led to significant policy interventions from federal and local governments. We leverage historical municipal expenditure data to construct estimates of recycling subsidies per tonne of waste and plot these (dotted red) against recycling rates (solid blue) in Figure 2 (Panel C).[12] Notably, the substantial increase in recycling rates during the decade between 1985 and 1995 coincides with a doubling of the estimated subsidies. Likewise, the slowdown in the rise of the recycling rate beginning around 1995 coincides with a stagnation in subsidies, which occurs simultaneously with the decline in the price of virgin materials (dashed black, Figure 2, Panel C). In the model developed in Section 3, profit-seeking firms will increase recycling rates in response to subsidies that cover a larger share of material recovery costs.

Taken together (Figure 2, Panels A-C), we view the data as strongly supporting the argument that circularity, and other material dynamics, are driven by economic forces. Producers substitute away from material use as the relative price of the input rises (price mechanism), and profit-maximizing recyclers increase recycling both in response to the price of their output and the subsidies they receive from the government (policy). The one important economic force absent in the data is technological progress. In the next section, we build a quantitative model of directed technical change in which the economic forces driving material use and circularity are directly linked to technological innovation.

3 A Growth Model of Material Use And Recycling

In this section we develop a model of growth that explicitly accounts for material flows into the economy, through the production and consumption stages, out of households as waste and then back to producers as recycled material. In Section 3.1 we develop the minimum theory needed to account for material flows in a dynamic economy and show that it can reasonably capture the data. In Section 3.2, we embed this structure in an endogenous growth model with directed technical change, which we use as the basis for our quantitative analysis. Finally, Section 3.3 presents a discussion about the static and balanced growth implications of scarcity for circularity.

3.1 Accounting for Aggregate Material Flows

Accounting for materials requires a stock-flow structure, which we develop using familiar concepts from growth models. Material flows can be summarized using four equations, namely, two empirical measures and two laws of motion.

First, we define total material use as the sum of virgin (V_t) and recycled (R_t) materials,

[12] See Appendix A.3.5 for details on the construction of the historical subsidies.

$M_t = V_t + R_t$. We then define the material share (of output) γ_t as

$$\gamma_t = \rho_t \frac{M_t}{Y_t}. \tag{1}$$

where the parameter ρ_t captures the share of materials that pass through the production process to be embodied in units of output. Note that $\rho_t < 1$ allows for material waste in the extraction or production process. Thus, γ_t is interpreted as the mass of materials, in tonnes, embodied by each unit of output which is then converted one-for-one into either consumption or investment.

The materials embodied in consumption are durable, so that they have their own law of motion. The stock of consumption follows

$$C_{t+1} = (1 - \delta_C)C_t + Q_t \tag{2}$$

where Q_t denotes period t purchases of consumption goods and δ_C is the consumption depreciation rate.[13] Combining the consumption law of motion with our measure of material share, we can derive the average material share in the current stock of consumption as

$$\bar{\gamma}_{t+1} = \left(\frac{Q_t}{C_{t+1}}\right)\gamma_t + \left(\frac{(1-\delta_C)C_t}{C_{t+1}}\right)\bar{\gamma}_t \qquad \textbf{(MLOM)} \tag{3}$$

We refer to this as the Materials Law of Motion (MLOM), which tracks the evolution of the average materials per unit of consumption stock. Note this is simply the weighted average of the materials share of all past purchases of consumption goods and is a sufficient statistic to compute how much material exists in the economy at any point in time.[14]

This formulation then provides a direct mapping to waste generation as measured in tonnes. The beginning-of-period t consumption stock (C_t) depreciates at rate δ_C, where each depreciated unit embodies $\bar{\gamma}_t$ tonnes of material. Thus, total waste generation in period

[13] In Section 3.2, we model a representative household who values consumption flow Q_t, keeping the framework as close as possible to a standard growth model. However, we treat materials embodied in consumption goods as durable and therefore must track the stock of consumption (C_t) for material purposes using the law of motion, equation (2). This allows for a more accurate mapping between measured inflows from MFA and outflows from EPA.

[14] By way of example, when a laptop is purchased, the consumer takes home both a collection of physical materials (plastics, metals etc), and all the embodied labor and capital in the product that deliver the services Q_t enjoyed by the consumer (software, production technologies etc). When the laptop breaks, the services disappear but the collection of physical materials remains. The materials law of motion (MLOM) we construct keeps track of how these materials (per unit of consumption or capital stock) evolve. This condition is closely related to, but different from, the statement that materials cannot appear or disappear out of nowhere, an equation generally referred to as the Second Law of Thermodynamics in the environmental literature; e.g., see Smith and Smith (1996).

t is computed as

$$W_t = \bar{\gamma}_t \delta_C C_t \tag{4}$$

Equations (1) - (4) provide a complete accounting structure for material flows in an economy, given a value of δ_C and time series for $\{M_t, Y_t, C_t, Q_t, W_t\}$. We construct our measure of M_t using data on virgin material flows from the United Nations MFA and recycling data from US EPA as described in Section 2.1. We obtain data on Y_t and Q_t directly from national accounts. We construct the consumption stock series C_t and calibrate the depreciation rate $\delta_C = 0.155$ following standard methods.[15] Finally, we rely on EPA for the time series W_t.

With these data in hand, Equation (4) directly yields a series of average material share per unit of consumption, i.e., $\{\bar{\gamma}_t\}_{t=1970}^{2015}$. Then, through the materials law of motion, Equation (3), we can compute the material share of output for each period t, i.e., $\{\gamma_t\}_{t=1970}^{2015}$. Finally, we obtain the series $\{\rho_t\}_{t=1970}^{2015}$ as the path of residuals needed to satisfy Equation (1), i.e. ρ_t adjusts so that the mapping between inflows (M_t) and outflows (W_t) balances every period. Ideally, the residual term ρ_t should be stable.

The left panel of Figure 3 plots the path of $\bar{\gamma}_t$, γ_t over the period 1980-2015, confirming that material shares have been declining over time. The right panel of Figure 3 plots the path of ρ_t over the same time horizon. We view the stability of ρ_t over this period as clear evidence that our framework can successfully account for aggregate material flows in the United States economy.

3.2 The Model

In this section we build an endogenous growth model that incorporates the materials accounting framework developed above and explicitly includes virgin material extraction and recycling choices. Reflecting existing government policies, we also include a role for taxes on virgin material use in production and subsidies to recycling. As is common in this literature, we adopt a directed technical change structure, so that material use and recovery technologies respond to scarcity over time.

3.2.1 Representative Household

We consider an infinitely lived representative household that derives utility from consumption flow (Q_t) and provides labor inelastically. Households rent labor and capital to the production sector, as well as R&D services in fixed supply D, which we assume are rented in their entirety each period. Given a sequence of labor, capital, and R&D resource prices, respectively, $\{w_t, i_t, p_{d,t}\}$, a lump-sum tax $\{T_t\}$, final goods, virgin and recycled materials

[15] See Appendix A.3.1 for details

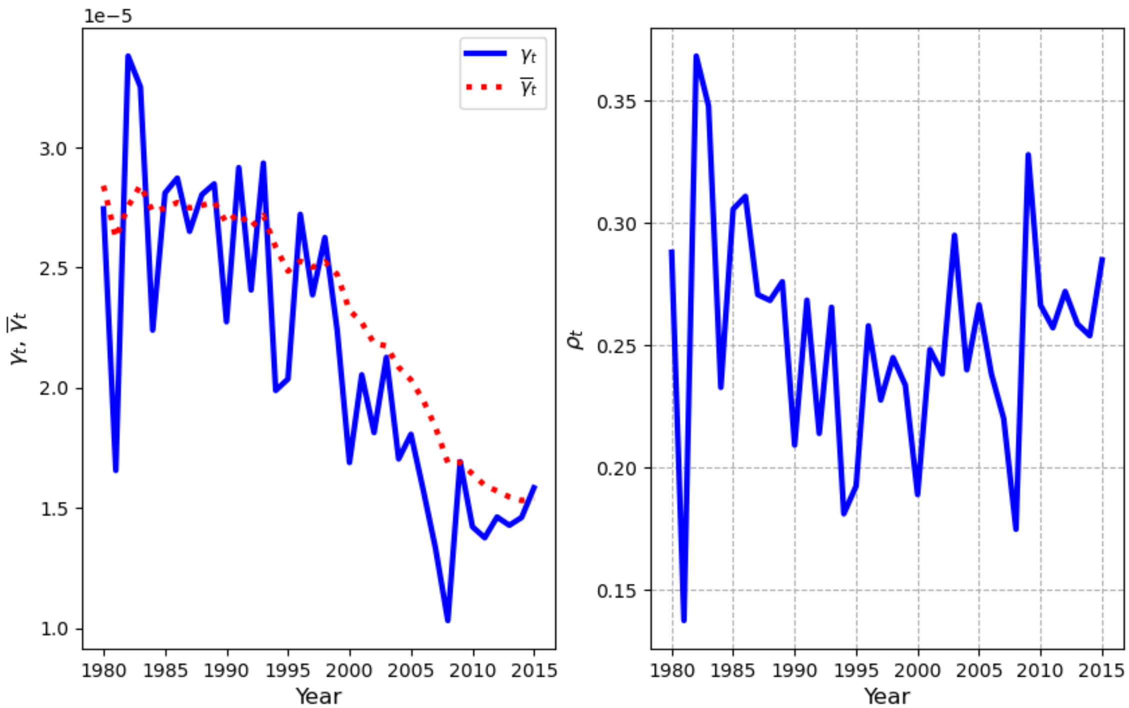

Figure 3: Material share γ_t (Equation 1), Average material share $\bar{\gamma}_t$ (Equation 3), and the share of materials ρ_t that pass through the production process to be embodied in units of output. Period 1980 - 2015.

producer profits $\{\pi_t^y, \pi_t^v, \pi_t^r\}$, respectively, and initial values for labor and capital $\{L_0, K_0\}$ the household solves:

$$\max_{X_t, Q_t, K_{t+1}} \quad \sum_{0}^{\infty} \beta^t u(Q_t)$$

$$Q_t + X_t \leq w_t L_t + i_t K_t + p_{d,t} D + \pi_t^y + \pi_t^v + \pi_t^r - T_t \tag{5}$$

$$K_{t+1} = (1 - \delta_K) K_t + X_t \tag{6}$$

$$L_{t+1} = g_n L_t \tag{7}$$

Equation (5) is the representative household budget constraint. Labor income, capital income, R&D income and profits, net of government transfers, are used to pay for investments in the capital stock (X_t) and purchase consumption (Q_t). Equation (6) represents the capital (K_t) law of motion, where δ_K is the capital depreciation rate. We assume the labor force grows at a constant gross rate g_n, Equation (7).

We next turn to producers in this economy. Production is divided into a final good producer (Y) and two primary materials producers: a virgin materials sector (V) and recycling sector (R). All sectors are competitive.

3.2.2 Virgin Primary Materials Producers

Within each period, competitive producers can extract virgin materials (V_t) at a fixed marginal cost denominated in the final good. As in Casey (2023), we assume that extraction costs are given by:

$$A_{v,t}\bar{V}_t^{\psi} \tag{8}$$

where $\bar{V}_t = \sum_{j=0}^{t-1} V_j$ is the sum of all materials so far extracted and $\bar{V}_{t+1} = \bar{V}_t + V_t$. The parameter $\psi > 0$ gives the elasticity of virgin materials extraction costs with respect to cumulative extraction and extraction costs fall with the productivity term $A_{v,t}$, which evolves exogenously.

The virgin materials producer, taking $\bar{V}_t$ and virgin materials price $p_{v,t}$ as given, solves

$$\pi_{v,t} = \max_{V_t} V_t \left(p_{v,t} - A_{v,t}\bar{V}_t^{\psi}\right) \tag{9}$$

The solution to this profit maximization problem implies that quantities are determined by virgin material demand and its price must equal the marginal cost. That is,

$$p_{v,t} = A_{v,t}\bar{V}_t^{\psi} \tag{10}$$

Notice that under this setup, virgin materials are in infinite supply but their extraction cost increases in cumulative use. We view this as a reasonable approximation given that we model a single material input - that is, we implicitly allow for substitution across all material types in the data. Rising extraction costs can then be thought of both as reflecting the need to obtain harder-to-extract materials as well as technical costs needed to allow for better substitution across materials.

3.2.3 Recycled Primary Materials Producers

A competitive recycling sector operates a production technology that converts waste to primary (recycled) materials (R_t) following:

$$R_t = A_{r,t}\left(W_t s_t\right)^{\alpha_r} \tag{11}$$

The recycler chooses effort (processing) s_t to increase recyclability, at proportional cost $\hat{c}_t = c_t - \tau_t$, where the unit cost c_t may receive government subsidies τ_t, also denominated in units of the final good.[16] Notice that while costs are in terms of consumption units, what matters for recycling quantities is the amount of embodied materials in the depreciated consumption $W_t = \bar{\gamma}_t \delta_C C_t$, equation (4).[17]

In addition to effort decisions, the recycler also has access to an investment technology that improves its TFP term $A_{r,t} = g_{A_r}(d_{r,t})A_{r,t-1}$, which is a function ($g_{A_r}$) of hired R&D resources ($d_{r,t}$), as discussed in Section 3.2.5. Hence, taking as given the recycled material price $p_{r,t}$, the R&D resource price $p_{d,t}$, the government policy τ_t and previous period TFP $A_{r,t-1}$, the recycler solves

$$\pi_{r,t} = \max_{s_t, d_{rt}} p_{r,t} A_{r,t} \left(W_t s_t\right)^{\alpha_r} - \hat{c}_t \left(\frac{W_t}{\bar{\gamma}_t}\right) s_t - p_{d,t} d_{r,t} \tag{12}$$

$$A_{r,t} \left(W_t s_t\right)^{\alpha_r} \leq W_t$$

$$A_{r,t} = g_{A_R}(d_{rt}) A_{r,t-1}$$

The recycler's optimal choices of s_t and $d_{r,t}$ must be such that no more than the material embodied in current waste can be recovered in any period, i.e., $A_{r,t} \left(W_t s_t\right)^{\alpha_r} \leq W_t$.

3.2.4 Final Goods Producer

Taking prices $w_t, i_t, p_{d,t}$ and a virgin material tax τ_v as given, along with initial TFPs $\{A_{n,t-1}, A_{m,t-1}\}$, the final goods producer solves the following profit maximization problem:

$$\max_{K_t, L_t, V_t, R_t, d_{n,t}, d_{m,t}} Y_t - w_t L_t - i_t K_t - (1+\tau_v) p_{v,t} V_t - p_{r,t} R_t - p_{d,t}(d_{n,t} + d_{m,t}). \tag{13}$$

Final goods (Y_t) are produced by combining capital and labor with a composite material aggregate $\tilde{M}_t$ according to the following production function

$$Y_t = \left[\delta_Y (A_{n,t} K_t^{\alpha} L_t^{1-\alpha})^{\frac{\epsilon-1}{\epsilon}} + (1-\delta_Y)(A_{m,t}\tilde{M}_t)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{14}$$

where $A_{n,t} = g_{A_n}(d_{n,t})A_{n,t-1}$ and $A_{m,t} = g_{A_m}(d_{m,t})A_{m,t-1}$ are resource-saving technologies for the capital-labor composite and the material composite, respectively. Both of these technologies can be improved through composite specific R&D investment $d_{n,t}$ and $d_{m,t}$, respectively. The parameter ϵ governs the elasticity of substitution between the capital-

[16] We abstract from capital and labor in this formulation because, in reality, recovery is a mix of government and private initiative, making measurement of factor inputs expended on recovery difficult to capture.

[17] This formulation reflects a well-recognized tension in the recycling and recovery world, often referred to as "lightweighting": as final goods producers use less materials through technological improvement, recovery costs per tonne of material rise (see, e.g., Cullen and Cooper (2022)). As a practical example, since the average can of soda has seen a decline in the quantity of aluminum used, the collection costs per ton of aluminum have risen. Compacting recycling is not feasible since materials must be sorted after collection.

labor composite and the materials composite and $\delta_Y \in (0,1)$ represents the share of the capital-labor composite in production.

The composite material aggregate $\tilde{M}_t$ is defined as

$$\tilde{M}_t = \left[(1-\delta_R)V_t^{\frac{\epsilon_m - 1}{\epsilon_m}} + \delta_R R_t^{\frac{\epsilon_m - 1}{\epsilon_m}}\right]^{\frac{\epsilon_m}{\epsilon_m - 1}} \tag{15}$$

where δ_R is the share parameter of recycled materials in the composite material aggregate and ϵ_m represents the elasticity of substitution between virgin and recycled materials.

3.2.5 Directed Technical Change

We allow for three sources of endogenous technology improvement $A_{j,t}$, $j \in \{n, m, r\}$, where n, m, r represent the capital-labor composite, the materials composite and recycled materials, respectively. Technology growth is modeled separately as $A_{j,t} = g_{A_j}(d_{j,t})A_{j,t-1}$ (Hassler et al., 2021; Casey, 2023), where

$$g_{A_j}(d_{j,t}) = \left(1 + \eta_j d_{jt}^{1-\lambda}\right), \quad \forall j \in \{n, m, r\}, \tag{16}$$

d_{jt} is the allocation of fixed factor $D = \sum_j d_{jt}$ to growing resource-saving technology j in period t, $\lambda \in (0,1)$ governs the decreasing returns to R&D within a period, and $\eta_j > 0$ represents the exogenous component of research efficiency. Assuming the extractive sector (virgin materials) technology $A_{v,t}$ evolves exogenously, all other resource-saving technologies $A_{n,t}$, $A_{m,t}$, and $A_{r,t}$ will be relative to this benchmark.

3.2.6 Government, Resource Constraint and Equilibrium

The government uses a lump sum tax and a resource extraction tax τ_v to finance the collection subsidy in the recycling sector. Hence, the government budget constraint is given by:

$$T_t + \tau_v V_t = \tau_t \left(\frac{W_t}{\bar{\gamma}_t}\right) s_t \tag{17}$$

The aggregate resource constraint implies then that output is equal to consumption and capital investment plus the resource cost of waste recovery and virgin material extraction.

$$Y_t = Q_t + X_t + \left(\frac{W_t}{\bar{\gamma}_t}\right) s_t c_t + V_t A_{v,t} \bar{V}_t^{\psi} \tag{18}$$

Optimality conditions and the full equilibrium definition are presented in Appendices A.2.1 - A.2.6. We now turn to some theoretical implications of the model.

3.3 Discussion

The model is designed to quantify how circularity responds to resource scarcity and policy in the presence of directed technical change with virgin extraction and recycling sectors. In this section we describe its short and long run implications.[18]

3.3.1 Optimal Innovation

In equilibrium, allocations of R&D resources yield a pair of expressions relating elasticities of innovation to relative expenditure shares:

$$\theta^Y_{KL}\frac{\eta_n d_n^{-\lambda}}{(1+\eta_n d_n^{1-\lambda})} = \theta^Y_M \frac{\eta_m d_m^{-\lambda}}{(1+\eta_m d_m^{1-\lambda})} \tag{19}$$

$$\frac{\eta_m d_m^{-\lambda}}{(1+\eta_m d_m^{1-\lambda})} = \theta^M_R \frac{\eta_m d_r^{-\lambda}}{(1+\eta_r d_r^{1-\lambda})} \tag{20}$$

where θ^Y_{KL} is the output share of total capital and labor expenditures, θ^Y_M is the output share of total material expenditure (adjusted for the profits earned on the recycling technology), and θ^M_R is the expenditure on material recovery as a share of total material spending. Thus innovation is greater where expenditure shares are relatively higher. In our quantitative exercises, these innovation responses to expenditure shares will have significant, long run impacts.

3.3.2 Scarcity, Policy and Circularity in the Short Run

Before treating the full growth model, we briefly discuss how resource scarcity, captured by total extraction $\bar{V}_t$, and policy impacts static virgin material use and recycling rates. For this analysis we take TFPs as given. We summarize these results in Proposition 1.

Proposition 1. *For static producer decisions*

1. *The material mix $\frac{R_t}{V_t}$ is increasing in scarcity $\bar{V}_t$, virgin tax and recycling subsidy.*
2. *The material share of output $\frac{\tilde{M}_t}{Y_t}$ is decreasing in scarcity $\bar{V}_t$ and the virgin tax, and increasing in the recycling subsidy.*
3. *The recycling rate is increasing in the recycling subsidy and ambiguous in scarcity, the virgin tax.*

Proof. See Appendix A.2.9 ■

[18] Complete derivations of the results presented in this section can be found in Appendix A.2.7

Combining optimal material choices from the final good's producer with the recycler's problem yields the optimal material mix

$$\frac{1-\delta_R}{\delta_R}\left(\frac{V_t}{R_t}\right)^{\frac{\epsilon_m-1}{\epsilon_m}}\frac{R_t}{S_{ut}}=\frac{(1+\tau_v)A_{v,t}\bar{V}_t^{\psi}}{\hat{c}} \tag{21}$$

An increase in scarcity, the virgin tax or the recycling subsidy all make recycled material relatively cheaper, raising the optimal material mix R_t/V_t. We show in Appendix A.2.7 that the material share responds in a similarly intuitive way. An increase in taxes or scarcity raises the cost of a unit of the material composite, while an increase in subsidies reduces it. Thus, the material share falls in scarcity and taxes while it rises in the subsidy.

Turning to the recycling rate, an increase in the subsidy will induce greater demand for materials and shift the optimal material mix up, leading to a higher recycling rate. Taxes or scarcity will have competing effects: material demand declines, reducing the demand for recycling, while the material mix rises. Which effect dominates is a quantitative question, depending on the levels of material in use and the degree of substitution between materials in the composite. In our quantitative model, taxes or scarcity will indeed increase recycling. Thus the model captures the basic intuition of a price mechanism driving circularity.

3.3.3 Balanced Growth Path (BGP)

In the long run, we are interested in a BGP of the economy, defined as a growth path where C, K, Y grow at constant, identical rates, all other aggregates and technologies grow at constant rates and the interest rate i_t and income shares are constant. The features of such a BGP are characterized below.

Proposition 2. *For any set of government policies τ, τ_v, labor growth rate g_n and growth rate of virgin extraction TFP g_{Av}, if a BGP exists then it is unique and:*

1. *Y_t, C_t, K_t, S_{ut} grow at rate $g_y = g_n g_{A_n}^{\frac{1}{1-\alpha}}$*

2. *$M_t, V_t, R_t, \bar{V}_t$ grow at rate $\left(\frac{g_y}{g_{A_v}}\right)^{\frac{1}{1+\psi}}$*

3. *$\gamma_t, \bar{\gamma}_t$ grow at rate $\left(g_y^{\psi} g_{A_v}\right)^{\frac{-1}{1+\psi}}$*

4. *Technology growth rates (and R&D allocations) are determined by the unique solution*

to the system of equations:

$$\begin{aligned} g_{Am}(d_m) &= g_{An}(d_n)^{\frac{\psi}{(1-\alpha)(1+\psi)}} g_n^{\frac{\psi}{1+\psi}} g_{Av}^{\frac{1}{1+\psi}} \\ g_{Ar}(d_r) &= g_{An}(d_n)^{\frac{1}{1+\psi}} \left(\frac{g_n}{g_{Av}}\right)^{\frac{1-\alpha_r}{1+\psi}} \\ 1 &= d_r + d_m + d_n \end{aligned} \tag{22}$$

Proof. See Appendix A.2.10. ■

Corollary 3. *Fixed government policy parameters τ, τ_v have no impact on BGP growth rates.*

Corollary 3 can be seen as a direct result of the BGP characterization in Appendix A.2.10. The result is in the same spirit as Casey (2023). In the long run, growth rates are driven by technical change. In turn, these rates respond to growth rates of relative prices. With constant taxes or subsidies, these terms net out and have no impact on long-run rates.[19]

3.3.4 Revisiting Scarcity, Policy and Circularity in the Long Run

Having characterized the long run dynamics of our model, it is worth revisiting the impacts of scarcity and policy on the economy's material share and recycling rate. We summarize these in Proposition 4

Proposition 4. *On the BGP*

1. *The material mix $\frac{R_t}{V_t}$ and material share of output $\frac{\tilde{M}_t}{Y_t}$ are independent of policies and initial scarcity $\bar{V}_0$.*

2. *The recycling rate is increasing in the recovery subsidy τ, decreasing in the virgin extraction tax τ_v, and independent of initial scarcity $\bar{V}_0$.*

Proof. See Appendix A.2.11. ■

A direct implication of Corollary 3 is that the paths of the material share and material mix on the BGP are independent of policy or initial scarcity $\bar{V}_0$. Growth rates are entirely determined by investment and other technologies, and these growth rates are what determine the ratios $\tilde{M}/Y$ and R/V. This is not to say policy has no impact: higher taxes, subsidies or greater scarcity will reduce the levels of material used, but not the growth paths.

The most striking result is that virgin material taxes unambiguously decrease the BGP recycling rate, in contrast to subsidies. It should be remembered that in the short run, the

[19]To change growth paths, the *growth rate* of taxes must be chosen, rather than the level. See discussion in Casey (2023).

impact of virgin taxes was unclear - although our calibration implies taxes boost recycling in the short run. To understand the long run result, it is instructive to rearrange the law of motion for the average material share in the consumption stock, Equation (3), substituting in BGP growth rates, to obtain

$$g_{\bar{\gamma}} g_y = 1 - \delta_c + \tilde{q} \frac{\rho M_t}{\bar{\gamma}_t C_t} \tag{23}$$

The left-hand side is the growth rate of the aggregate mass of materials in the consumption stock. As noted by Corollary 3, this is independent of policy. The right-hand side has a depreciation component and then a flow-component, which is the product of two terms. We define $\tilde{q} = \frac{Q_t}{Y_t}$ as the BGP consumption share of output. The term $\frac{\rho M_t}{\bar{\gamma}_t C_t}$ is the current materials share of the total mass of materials in the consumption stock. Note however that on the BGP, the recycling rate can be expressed as $RR = \frac{(1-\tilde{v})M_t}{\delta_C \bar{\gamma}_{t-1} C_{t-1}}$, which is proportional to this ratio. Thus Equation (23) implies that on the BGP, $\tilde{q}$ and the recycling rate must be negatively related.

When the virgin extraction tax τ_v is increased, this induces less extraction to occur. In the long run, this frees up aggregate resources for other things, including consumption, leading to a rise in $\tilde{q}$. However, if a larger share of output flows into consumption, it must be that each unit of consumption embodies a smaller share of materials relative to the existing stock in order for overall growth in material mass to remain constant. In turn, this means a relatively smaller share of materials are used, including recycled materials, implying a lower recycling rate. This result depends critically on the measurement of consumption-based recycling only and in our view is most useful as a caution against over-reliance on MSW-based measures of recycling as guidance towards circularity.

4 Quantifying the Costs of Circularity

The results presented in the previous section have mostly focused on short- and long-run analytical outcomes. To investigate model implications quantitatively, we first need to calibrate it. Once the model is calibrated to the US, we study the quantitative implications of our baseline parameters for circularity in the US economy. Then, we perform several exercises to fully explore the interactions between endogenous innovation and material use policies.

4.1 Calibration

In this section we discuss how we map the model to data. We calibrate the preferences, production and innovation components of the model in two steps. In the first step, we calibrate a group of parameters directly from the data series. In the second step, we jointly calibrate the remaining R&D parameters. Table I presents our baseline parameters for the US economy.

External Parameters and Data. To match our annual data on material use and recycling, we set the model period to one year. We then follow the literature to externally set standard values for the parameters β, σ and α. The discount rate is set to $\beta = 0.96$, the relative risk aversion coefficient is set to $\sigma = 2.00$, and the capital share in the final goods production is $\alpha = 0.35$ (Golosov et al., 2014). The average growth rate of the labor force (BLS Employment) over the period 1980-2015 is approximately 1.29%, which implies $g_n = 1.01$.

Direct Calibration. Parameters related to the laws of motion of consumption, capital and materials (material flow accounting), i.e., δ_C, δ_K, and ρ are calibrated according to standard methods (Section 3.1 and Appendix A.3.1). The consumption and capital depreciation rates are respectively 15.5 and 10.7 percent and we set $\delta_C = 0.16$ and $\delta_K = 0.11$, respectively. We set $\rho = 0.25$, which is the average value from our materials accounting exercise between 1980-2015 (Figure 3, right panel).

Profit maximization of the virgin materials producer implies $p_{v,t} = A_{v,t}\bar{V}_t^{\psi}$. We construct a measure for $\bar{V}_t$ (sum of all materials extracted up to period t) based on the material extraction (DE) data available from Krausmann et al. (2018). The price equation, along with data on the historical prices of virgin materials $p_{v,t}$ and $\bar{V}_t$, allows us to recover ψ (Appendix A.3.3). As a benchmark, we set the elasticity of material prices to cumulative material extraction as $\psi = 1.12$. With our calibrated value of ψ, we construct a series of virgin materials TFP $A_{v,t}$, which yields an average growth rate over the period 1980-2015 of -3.1% ($g_{A_v} = 0.97$, Appendix A.3.9).

For the parameters governing the recycled materials producer problem $(\delta_R, \alpha_R, \varepsilon_m)$, we first set $\delta_R = p_{rt}R_t/(p_{vt}V_t + p_{rt}R_t) = 0.01$ to match the expenditure share of recycled materials in the US economy (Appendix A.3.4). The solution of the recycler's problem (ignoring DTC) implies the following representation of recycled materials:

$$R_t = A_{rt}^{\frac{1}{1-\alpha_r}} \left(\frac{\bar{\gamma}_t p_{rt} \alpha_r}{\hat{c}_t} \right)^{\frac{\alpha_r}{1-\alpha_r}}$$

where $\hat{c}_t = \bar{\gamma}_{t-1}(c_t - \tau_t)$ (Appendix A.3.5). Taking logs of the growth rate of recycled materials (R_{t+1}/R_t) and estimating the equation, we recover a calibrated recycler's production function parameter α_R equal to 0.31. Combining the first-order conditions of the final goods producer problem with respect to V_t and R_t (Appendix A.3.6), we have

$$\ln\left(\frac{V_t}{R_t}\right) = \left(\frac{\epsilon_m}{\epsilon_m - 1}\right)\ln\left(\frac{p_{vt}}{p_{rt}}\frac{\delta_R}{(1-\delta_R)}\right)$$

Estimating this equation, we obtain $(\epsilon_m/(\epsilon_m - 1)) = -1.0522$, which implies $\epsilon_m = 0.51$.

For final goods production, equation (14), we set the share of the capital-labor composite in production to $\delta_Y = 0.88$ to match the average material expenditure share in the data, computed as $(p_{vt}V_t + p_{rt}R_t)/Y_t$ (Appendix A.3.7). And, finally, following Hassler et al. (2021), we estimate ε with a maximum-likelihood approach, which yields $\varepsilon = 0.26$ as our benchmark value for the elasticity (Appendix A.3.8).

R&D Internal Calibration. The R&D production functions (Section 3.2.5) have four unknown parameters, η_n, η_m, η_r and λ. The η terms capture the inherent efficiency of R&D in improving the three types of technology (capital-labor composite, the materials composite and recycled materials, respectively), while λ governs the degree of diminishing returns. A common approach in the literature is to assume the economy is on a balanced growth path, in which case these parameters can be read off either directly from data (e.g. Casey, 2023) or from model responses to price shocks (e.g. Fried, 2018). However, the growing recycling rate (Figure 1) and material mix are inconsistent with balanced growth in our model. Instead, we initialize the economy in 1980 in intensive form (Appendix A.3.10), searching for the R&D parameters to reproduce features of the model economy during the 1980-2015 time period.[20]

The model is sharp in terms of what is required for the parameter values of the R&D production functions. Output growth demands a low efficiency of the capital-labor technology, i.e., low η_n. This, in turn, constrains the efficiency of material composite technology η_m, in order to deliver the appropriate decline in the material-output (M/Y) ratio. Parameters η_r and λ are jointly pinned down by the recycling rate and growth rate of A_r. There are a locus of λ, η_r pairs that deliver the path of the recycling rate, but with markedly different implications for the convexity of the A_r path. Matching these simultaneously delivers our calibrated values.

Internal calibration yields the values reported in Table I. We find diminishing returns of $\lambda = 0.45$ and capital-labor, materials composite and recycling R&D efficiency parameters of $\eta_n = 0.01$, $\eta_m = 0.05$, and $\eta_r = 0.21$, respectively. The results suggest that improving recycled materials efficiency technology is inherently easier than improving the material

[20]For the calibration, we use a version of the model in which virgin material prices are exogenous, feeding in the exact path of prices and recycling subsidies that we compute in Section 2 (Appendix A.3.5). This approach avoids the concern of whether US virgin extraction alone can deliver the material price path observed in the data (particularly the 2000s commodity boom).

Table I: Calibration Results

Description		Parameter	Value
		<u>External Calibration</u>[1]	
Subjective discount rate		β	0.96
Relative risk aversion coefficient		σ	2.00
Production function capital share		α	0.35
Population growth (%)		g_n	1.01
		<u>Direct Calibration</u>[2]	
Consumption, Capital, Materials Laws of Motion			
Consumption depreciation rate		δ_C	0.16
Capital depreciation rate		δ_K	0.11
Fraction of materials extracted		ρ	0.25
Virgin Materials Producer			
Elasticity virgin materials extraction		ψ	1.12
Virgin's TFP growth rate		g_{A_v}	0.97
Recycled Materials Producer			
Recycled materials relative weight		δ_R	0.01
Recycler's production function parameter		α_R	0.31
Elasticity of substitution materials composite		ε_m	0.51
Final Goods Producer			
Materials expenditure share		δ_Y	0.88
Elasticity of substitution KL-materials		ε	0.26
		<u>Internal Calibration</u>[3]	
Directed Technical Change			
Research decreasing returns parameter		λ	0.45
Research Efficiency:	Capital-Labor	η_n	0.01
	Materials Composite	η_m	0.05
	Recycled Materials	η_r	0.21

Notes: (1) α: Golosov et al. (2014); g_n: BLS Employment; (2) Appendices A.3.1 - A.3.9; (3) Appendix A.3.10

composite technology. As the data show, only a small share of overall material use is from recycling, which requires the investment efficiency of recycling TFP (η_r) to be more than four times that of the materials efficiency η_m in order to justify investment in A_r, as well as a growing recycling rate. Intuitively, if the relative benefit of investing in recycling efficiency is low, but the growth rate of the recycling rate is high, it must be the case that the cost of improving recycling efficiency is relatively low. Similarly, the investment efficiency of materials (η_m) is nearly six times that of the capital-labor composite (η_n), which is required to rationalize investment in materials efficiency when the overall material share is small relative to the capital-labor composite.

Model and data moments. Table II reports model and data moments. The target moments are the recycling rate (RR), the growth rate of output (g_y), the materials-output ratio (M/Y) and the recycler's TFP growth rate (g_{A_r}). As some additional model validation, Table II also reports three other non-target material dynamics over the 1980-2015 period: material mix (R/V), the virgin material-output ratio ($p_v V/Y$) and the capital-output ratio (K/Y). The ratios RR and M/Y, R/V, $p_v V/Y$ are reported as the average percentage point change and the variables g_y, g_{A_r},K/Y are reported as the average over the period 1980-2015.

Table II: Targeted and Non-Targeted Moments

	Model	Data
Target Moments		
Recycling Rate (RR)	6.55	6.09
Output Growth Rate (g_y)	2.67	2.59
Material-Output Ratio (M/Y)	-0.98	-1.09
Recycler's TFP Growth Rate (g_{A_r})	5.03	5.61
Non-Target Moments		
Material Mix (R/V)	1.02	1.23
Virgin Material-Output Ratio ($p_v V/Y$)	-2.28	-2.32
Capital-Output Ratio (K/Y)	1.50	1.38

Notes: The ratios RR, M/Y, R/V, $p_v V/Y$ are reported as the average percentage point change, i.e., $(X_{2015} - X_{1980})/35$. The variables g_y, g_{A_r},K/Y are reported as the average over the period 1980-2015; g_y, g_{A_r} reported in percentage (%) terms. Magnitudes: $RR, R/V \times 10^{-3}$ and $M/Y, p_v V/Y \times 10^{-6}$.

As one of the key measures of circularity, Figure 4 reproduces recycling rates (RR) implied by the model (red dotted line) and observed in the data (blue solid line). Also reported in Figure 4 are the material mix (R/V), the virgin material-output ratio (M/Y) and the recycler's TFP (A_r). The material mix increases by 4.3 percentage points in the data and 3.6 percentage points in the model and the virgin material-output ratio (recycler's TFP) falls (increases) by approximately half (by factor of four) in both the data and model. In addition, the model performs well regarding the average growth rates of the non-targeted TFPs A_m and A_n: 2.77% (model) versus 2.69% (data) and 0.54% (model) versus 0.71% (data), respectively.

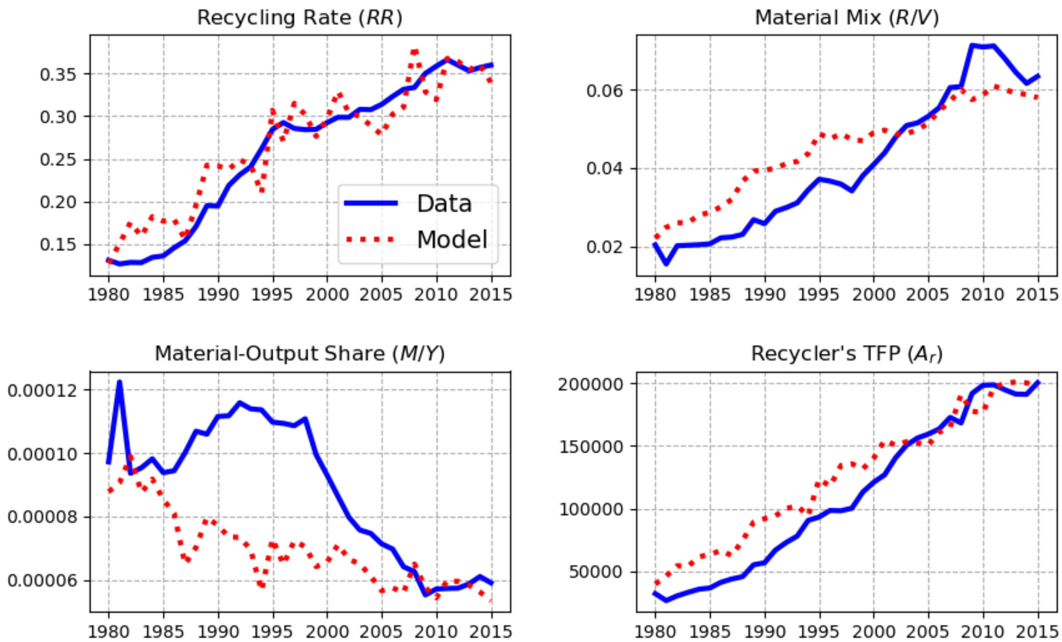

Figure 4: Recycling Rate (RR), Material Mix (R/V), Material-Output Ratio (M/Y) and Recycler's TFP (A_r), Model and Data

4.2 The Drivers of US Circularity: 1980-2015

We begin our quantitative analysis by studying how much existing policy has contributed to the historical path of circularity in the US. In the process, we develop useful intuition for the key role of technological change in the model.

Figure 5 presents de-trended recycling rates in the benchmark calibration and then in three counterfactual exercises that hold each of the subsidy policy, virgin material prices or recycling technology constant. Holding the recycling subsidy constant at its 1980 value (black dashed line, Figure 5), before the substantial expansion of recycling programs in the US, the model finds the recycling rate would have been four percentage points lower (31%).Thus we find that the expansion of recycling subsidies in the 1980s made a modest contribution to the overall path of circularity in the US. We also find that the lower virgin material prices of the 2000s did indeed play a role in the flattening recycling rate, as suggested by Figure 2 (Panel C). Keeping prices constant at their 1980 level (red dotted line, Figure 5), which avoids this period of lower prices, the model implies the recycling rate would have been three percentage points higher in 2015 (at 38%). However, the primary lesson that emerges from Figure 5 is the pre-eminent role of recycling technology. Absent improvements in A_r, the US recycling rate would have fallen to 10% by 2015.

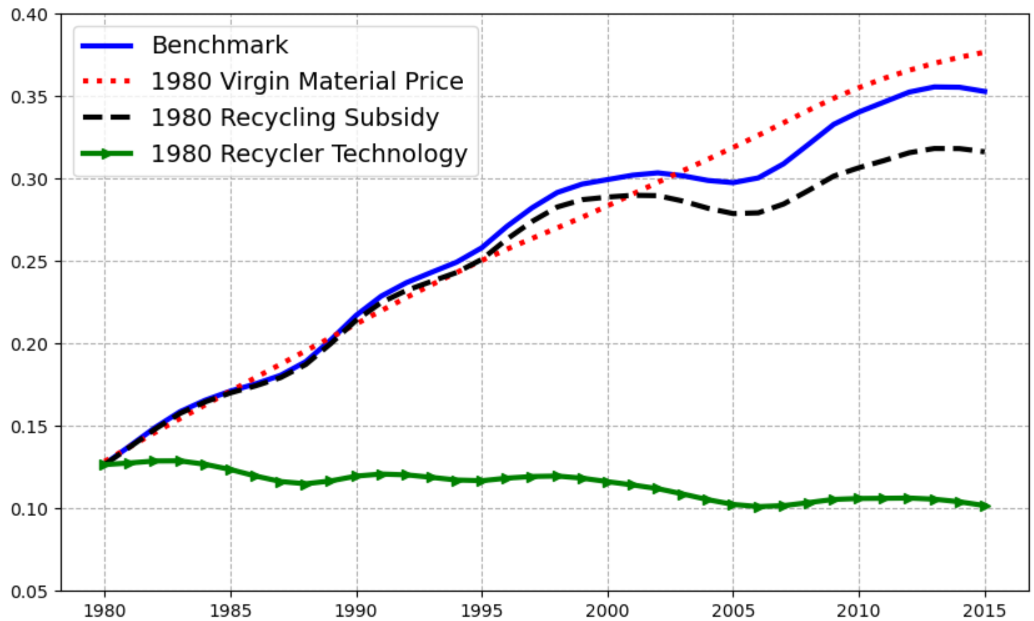

Figure 5: Decomposition Exercise - Recycling Rate (De-trended Series), 1980-2015.

Given the evident importance of recycling TFP that we observe in Figure 5, it is instructive to study the endogenous response of technologies in these various counterfactuals. Two lessons emerge. First, subsidies reduce investment in material technologies A_r and A_m, as material expenditures fall. By 2015, A_r is a full 6% higher in the counterfactual without subsidies (relative to benchmark), and investment in the capital-labor TFP declines (Column 1, Table III). So, technology responses dampen the impact of subsidies on recycling. Second, if we compare the constant price counterfactual to the benchmark, we see that during the commodity boom of the mid-1980s to mid-1990s, innovation responds by moving towards material technologies and away from capital-labor TFP (Column 2, Table III). As prices begin to decline and material expenditures follow suit, innovation shifts back towards capital-labor TFP (Column 3, Table III). The small size of these changes is a result of the short time horizon studied. In the next section, we will see that these dynamics compound over longer time horizons as we evaluate the costs of achieving the stated U.S. target of a 50% recycling rate by 2030.

4.3 Achieving an Exogenous Recycling Rate Target

In our benchmark model with endogenous prices, the US recycling target of 50% by 2030 is out of reach absent further policy intervention (blue solid line, Figure 6). The recycling

Table III: Counterfactuals - Relative Change

	I	II	III
	Low Subsidies (1980-2015)	High Prices (1985-1997)	Low Prices (1997-2015)
ΔA_n	0.9991	0.9985	1.0036

Notes: Counterfactual I: $\Delta A_n = A_n^{\tau_{1980}}/A_n^{Bench}$; Counterfactual II: $A_n^{Bench}/A_n^{p_v,1980}$; and Counterfactual III: $A_n^{Bench}/A_n^{p_v,1980}$.

rate falls below 35% by 2030 as virgin material prices decline. While we find the need for policy is generally robust to alternative assumptions, the exact path of recycling is heavily influenced by price paths (in Section 4.5, where we consider an increasing, exogenous virgin material price, the recycling rate remains above 35% out to 2100).

In this section, we consider two policy alternatives to increase the recycling rate and study their implications, in particular, how costly will it be to achieve the EPA target. Because we want to allow for long run impacts of material scarcity, we use our benchmark model with endogenous prices. As proposed in the US, we focus on a subsidy to the recycling cost, as presented in the recycled primary materials producers problem (equation 12). Alternatively, and as proposed in the EU, we also consider a virgin material tax (final goods producer, equation 13). In both cases, we consider a permanent, one-time increase in the policy lever in 2024, holding all other policy constant.

Achieving the US recycling target through subsidies requires a doubling of the current subsidy, from covering 42% of costs in 2015 to 87% of costs in 2024. Notice that this target is reached immediately - policy is implemented in 2024 and the target is achieved in 2025 (black dashed line, Figure 6). A higher recycling subsidy increases the recycling rate on impact and decreases the relative price of recycled material, leading to a similar jump in the material mix R/V. Measuring the cost of this policy in consumption-equivalent terms relative to the benchmark, Table IV shows that the recycling subsidy is not very costly, implying a 0.17% decline in welfare. It also leads to an increase in output growth of 1/100th a percent, which by 2100 implies a 0.22% increase in output relative to the benchmark economy.

By contrast, achieving the recycling target through a virgin recycling tax is extremely costly, as it only indirectly impacts the recycling rate (red dotted line, Figure 6). To see this, we compute a welfare-cost-equivalent virgin tax and study its impact on recycling dynamics. Imposing an 83.5% tax on virgin material delivers the same welfare costs as the 87% subsidy above, but yields a maximum recycling rate of slightly above 40% in 2033. These slower dynamics are because the virgin material tax largely operates by decreasing the material share γ_t, which feeds into the average material share $\bar{\gamma}_t$ over time and pushes the recycling

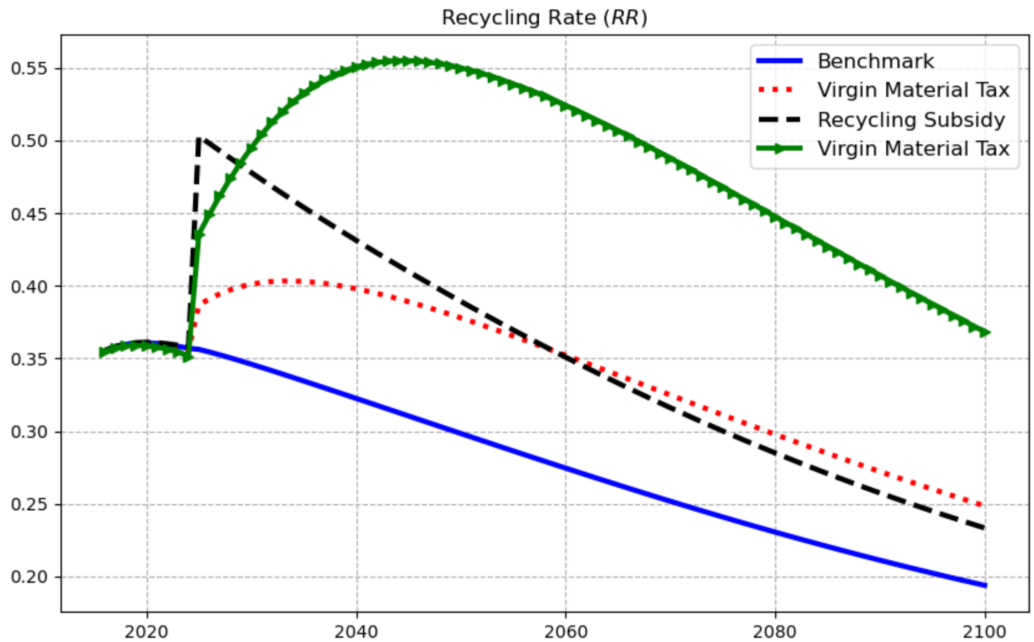

Figure 6: Recycling Rate (RR) - Benchmark, Virgin Material Tax (Equal Cost), Recycling Subsidy, 1980-2100.

rate up. While welfare costs are the same, output dynamics are different from the subsidy. Output growth falls by 2/100ths of a percent and by 2100 this yields a 1.77% decline in output relative to the benchmark.

Thus, our initial estimates suggest that achieving the stated U.S. target of 50% recycling by 2030 is not only feasible but relatively inexpensive when subsidies are used, as indeed U.S. policy is doing. Subsidies must double, and this comes at a cost of slightly less than 2/10ths of a percent of welfare in consumption-equivalent terms. Even if we instead focus on the EU circularity measure, the material mix R/V, the subsidy is a more efficient policy tool and we can obtain a 2.5 percentage point increase in the material mix (increasing the overall rate by nearly half) through the same subsidy policy.

4.4 Targeting Less Virgin Material Extraction

Our findings in Section 4.3 suggest that achieving U.S. circularity targets will be relatively inexpensive in welfare terms. But the circularity dynamics analyzed above, where both the recycling rate and material mix are ratios, mask changes in levels that seem undesirable. In response to subsidies, virgin material extraction actually rises above extraction levels in the benchmark economy. This is because subsidies reduce the price of recycled material, which our calibration finds are weak complements with virgin material. As the overall price

Table IV: Policy Experiments - Recycling Subsidy, Virgin Material Tax, Virgin Extraction

	I	II	III	IV
	Benchmark	Recycling Subsidy	Virgin Tax Equal Cost	Virgin Extraction
Average g_y (%)	2.48	2.49	2.46	2.39
$Y_{Policy}/Y_{Benchmark}$	-	0.22	-1.77	-7.22
$CE_{Policy}/CE_{Benchmark}$	-	-0.17	-0.17	-1.42

Notes: Average g_y is the average over the period 2015-2100. The variable $Y_{Policy}/Y_{Benchmark}$ reports the change in output in 2100 relative to the benchmark level in 2100. The variable $CE_{Policy}/CE_{Benchmark}$ reports the percentage change in the CE due to the policy (CE_{Policy}) relative to the benchmark CE ($CE_{Benchmark}$)

of the material composite falls, this pushes total material use up, rather than down. If the concern underpinning the calls for a more circular economy is to reduce stress on the natural environment through extractive activity, the U.S. subsidy policy may be pushing the economy in the wrong direction.

These dynamics can be seen in Figure 7, which plots the paths of virgin material extraction (relative to the benchmark) in the cases of the subsidy (black dashed line) and cost-equivalent tax (red dotted line) studied in Section 4.3. In response to the subsidy, virgin extraction initially falls relative to the benchmark, however by 2080 virgin extraction begins to exceed that of the benchmark economy. By comparison, the virgin material tax (equal cost) leads to an instantaneous decline in virgin extraction of 17% relative to the benchmark, which grows to a 27% decline by 2100.

These results suggest that achieving a substantial reduction in levels of material extraction, as opposed to recycling rates or the material mix, may in fact be much costlier. To put this in context, we compute a virgin material tax that reduces virgin extraction in 2024 to the level of 1970. Such a policy goal requires an enormous tax on virgin materials of four-hundred percent (397%). This leads to an immediate reduction in virgin extraction on the order of 40% (green arrow line, Figure 7), growing to a 61% reduction in virgin extraction (relative to the benchmark) in 2100. Incidentally, this policy achieves a 50% recycling rate by 2030 and maintains a recycling rate above the 2015 level out through 2100 (green arrow line, Figure 6), while the material mix nearly doubles. However, such a reduction in virgin extraction comes at enormous cost. Table IV shows that this policy leads to a 1.42% decline in consumption equivalent welfare, which understates the long-run costs since the gap in consumption relative to the benchmark grows over time. By 2100, aggregate output is 7.22% lower under this policy and consumption lower by 5.8%. The increase in material expenditure arising from the tax leads to a shift in innovation towards material technolo-

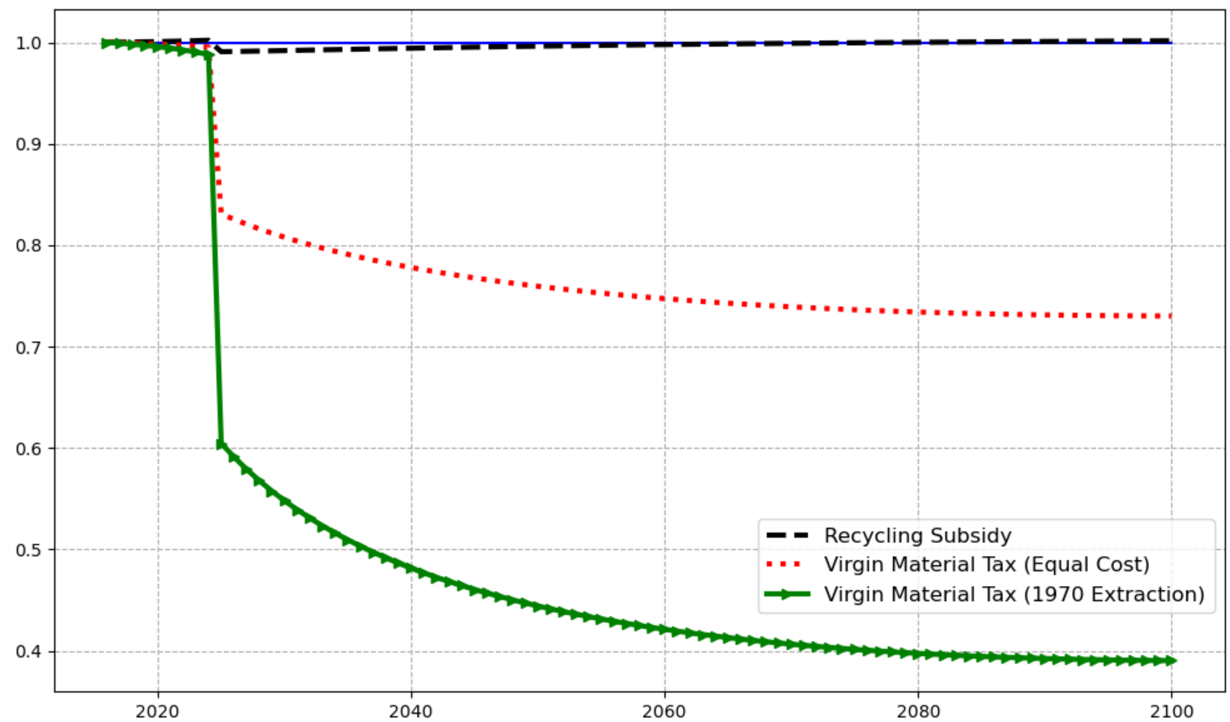

Figure 7: Virgin Material Extraction (V) - Recycling Subsidy, Virgin Material Tax (Equal Cost) - Policy Relative to Benchmark, 2015-2100.

gies and away from the capital-labor TFP, implying a decline in output growth of 1.1% per decade.

4.5 Sensitivity Analysis

The discussion of endogenous innovation responses in Section 4.2 made clear that prices are important for the direction of innovation and material use. In the calibrated model with endogenous prices, which was used for computing the costs of achieving various material use targets, virgin material prices decline at approximately 1% annually from 2015 to 2100. This is less than the roughly 2% annualized decline in material prices realized in the data between 1995 and 2015, but we may be concerned about the idea that U.S. extraction alone drives material prices, an assumption implicit in the endogenous price model. For robustness, rather than directly changing ψ we consider the exogenous price model where we assume that prices grow slowly, at 1% per decade, starting in 2015.[21] Figure 9 (left panel) compares recycling rates from 2015-2100 between the exogenous- and endogenous-price models.

With no policy changes, a slowly growing (exogenous) price as opposed to a falling (endogenous) price has very different implications for recycling in the short-to-medium run.

[21] The value $\psi = 1.12$ that we estimate is similar to that found by Casey (2023) who, studying energy use, estimates $\psi = 1.26$ as the elasticity of energy extraction costs with respect to cumulative extraction.

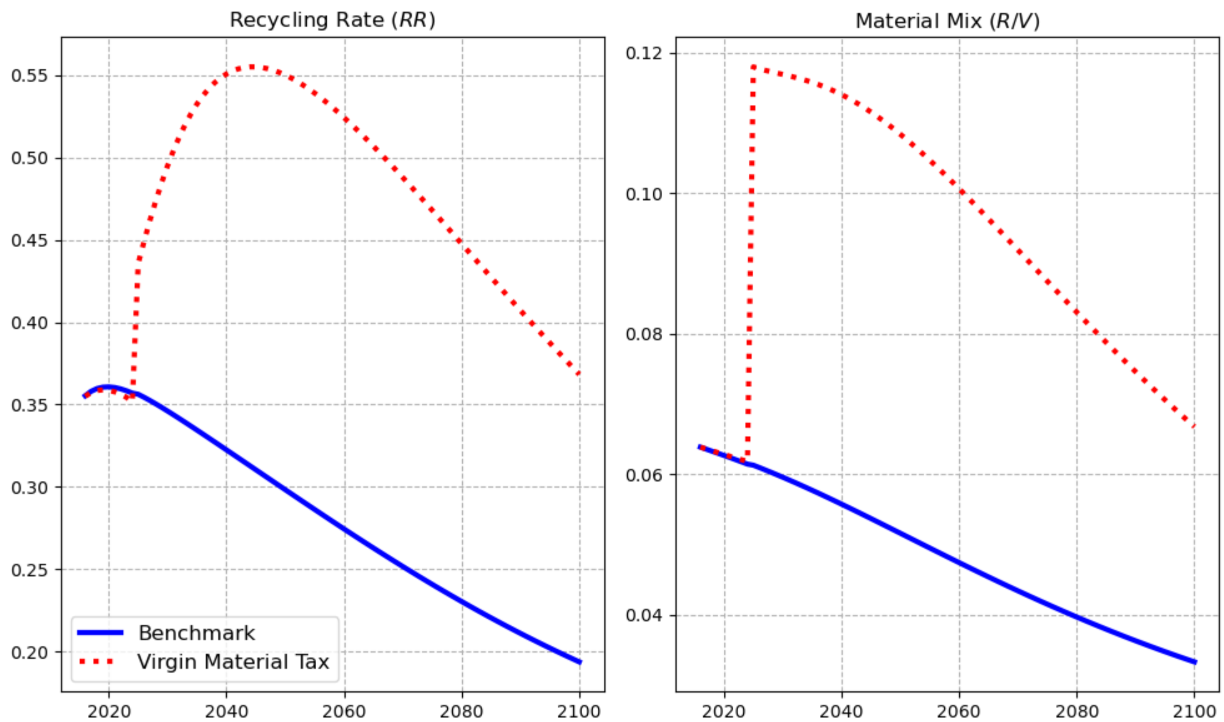

Figure 8: Recycling Rate (RR) and Material Mix (R/V) - Benchmark and Virgin Material Tax (1970 Virgin Material Extraction Level), 1980-2100.

By 2100, the recycling rate is almost 15 percentage points higher, although beginning to fall. We can see how higher prices, in the exogenous price case, drive technical change by comparing annualized TFP growth rates in the two models. With exogenously growing prices, recycling TFP (A_r) grows at 1.29% as compared to 1.14% in the endogenous price case. The material composite TFP (A_m) grows at 0.98% as compared to 0.66%, and this comes at the cost of growing the capital-labor composite TFP (A_n), which slows down from growing at 0.68% in the endogenous price economy to 0.66%.

In spite of these differences however, the policy implications with growing prices are surprisingly robust. We recompute the subsidy required to obtain a 50% recycling rate by 2030 and find this requires a subsidy covering 84% of costs, as opposed to 87% in the endogenous price model. We also compute the virgin material tax required to obtain the 1970 level of extraction in 2024 and find that this requires a tax of 316% as opposed to 397% with endogenous prices. Thus policy implications are quite similar across different paths of material prices.

It is worth noting that, while policy implications are similar, the longer run implications across different prices scenarios do differ. With exogenous, growing prices, the virgin material tax delivers a peak recycling rate of 67% and remains well above 60% in 2100. With

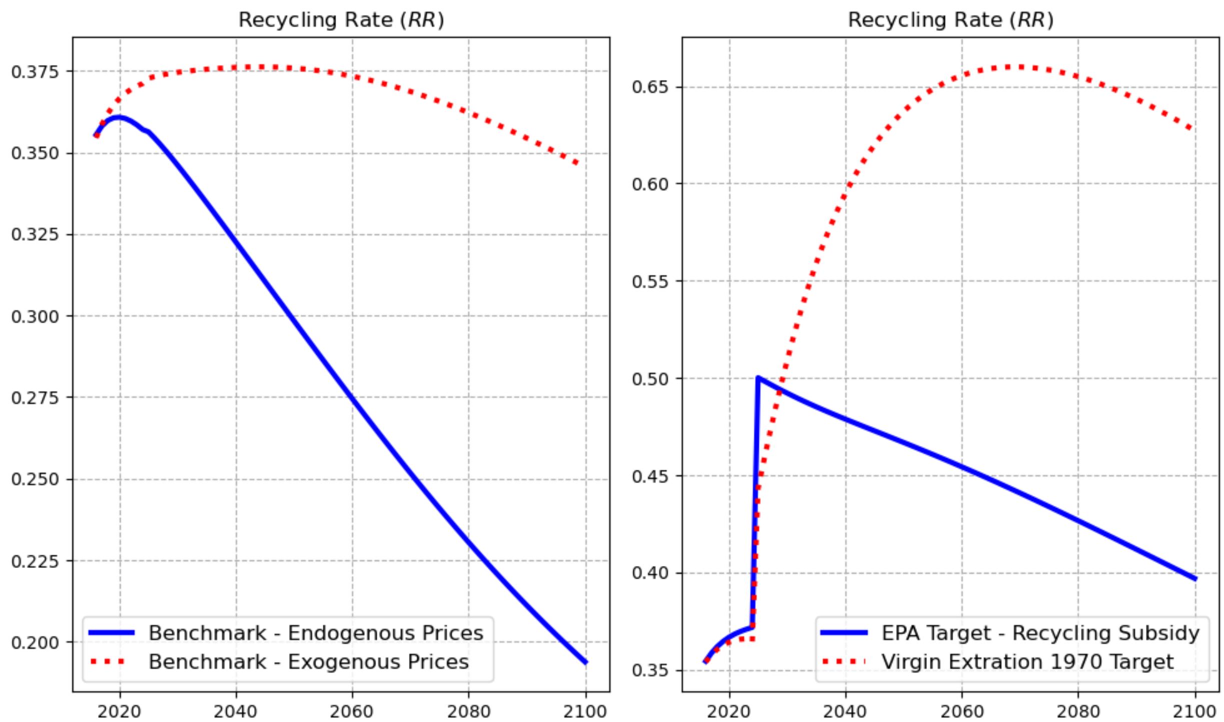

Figure 9: Recycling Rates: Endogenous v. Exogenous Prices (left panel) and Policy Responses Under Exogenous Prices (right panel)

endogenous, declining prices, an even larger virgin material tax yields a peak recycling rate of 50% and has fallen to about 40% by 2100 (right panel, Figure 9).

We also conduct sensitivity analysis for other key model parameters. We vary all four parameters governing innovation[22] (λ, η_n, η_m, and η_r) and for each case compute the subsidy required to obtain a 50% recycling rate by 2030. The results are insensitive, with the recycling subsidy required to reach the EPA target ranging from 83% to 89% (benchmark: 87%).[23].

In addition to the internally-calibrated parameters governing technical change, material dynamics are particularly influenced by ϵ_m, the elasticity of substitution between virgin and recycled materials in the material composite $\tilde{M}_t$. Our benchmark estimate of $\epsilon_m = 0.513$ suggests virgin and recycled materials are complements, and relatively more substitutable between each other than materials are with the capital-labor composite ($\epsilon = 0.255$). One concern is that, in order to obtain a long series of recycled material prices, we proxy for the growth in recycled prices using virgin material price indices (see Appendix A.1.1). As a

[22]We consider two alternative λ values, i.e., $\lambda = 0.40$ (Casey, 2023) and $\lambda = 0.50$ (Acemoglu et al., 2018; Akcigit and Kerr, 2018), which are about ten percent lower and higher than our benchmark value ($\lambda = 0.45$), respectively. In the same fashion, we consider research efficiency parameters that are ten percent lower and higher than our benchmark values $\eta_n = 0.007$, $\eta_m = 0.045$, and $\eta_r = 0.205$ (Table I).

[23]See Table A.5, Appendix A.6 for details

robustness check, we re-run our estimation of ϵ_m using historical spot prices from recycling markets, which are available only from 1990. On this shortened sample of 25 years, we find $\epsilon_m = 0.4832$, which suggests our benchmark value is reasonable.

5 Conclusions

Concerns about the environmental impacts of growing raw material extraction and waste generation have led to calls to reduce the extraction of new materials from the earth and boost material recycling, i.e. improve economic circularity. Building on the directed technical change literature, this paper develops a quantitative growth model that includes virgin and recycled material sectors. We show that the model can accurately account for material flows in the data and can reproduce key historical patterns of material use and recycling in the US. The calibrated model can be used to evaluate a wide range of policies and we provide the first estimates of the macroeconomic costs of US circularity targets.

A key insight of this paper is the need to understand material dynamics through the lens of the profit-maximizing behaviour of final goods producers and material recyclers and the allocation process of scarce innovation resources toward material (virgin, recycled) and non-material (capital, labor) inputs. Introducing two data sources (MFAs, EPA) to the macroeconomic literature, we present empirical evidence that highlights the increasing circularity in the US over the past 50 years, with rising relative material prices and government policy interventions playing significant roles. The US economy has produced more output per tonne of material used, has shifted toward the use of recycled materials and is recycling an increasing share of waste materials for reuse. While the focus on materials demands a specific production structure and material flow accounting absent in DTC models of energy use, we also consider the important role for endogenous, resource-saving technology responses to policies. The model captures final goods producers and material recyclers profit motives and can reproduce well the historical paths of material use and recycling, which are strongly influenced by the virgin material price.

Using our calibrated model, we provide sharp, quantitative estimates of the cost of obtaining US circularity targets and contrast these with the cost required to substantially reduce the US economy's reliance on the extraction of new material from the earth. Achieving ambitious recycling targets, such as the EPA's goal of a 50% recycling rate by 2030, requires a doubling of the current subsidy but is relatively inexpensive in terms of consumption-equivalent welfare. Alternatively, using a virgin material tax (as proposed by the EU) to achieve the same target turns out to be much more costly. We need to impose a high tax on virgin material to deliver the same welfare costs as the subsidy, which leads output to decline relative to the benchmark. While subsidy policies are inexpensive ways to boost the recycling rate, we find they actually increase material extraction in the medium run. This is because we estimate recycled and virgin materials to be weak complements, so recycling subsidies actually encourage greater material use. Policies intended to reduce material extraction itself are quite costly. Returning to 1970 levels of extraction (a reduction on the order of 40% relative to current levels) would require an extremely high tax on virgin material use, leading to a substantial decline in consumption-equivalent welfare.

There is significant scope for incorporating physical material flows into quantitative macro models, leveraging MFAs and other data sources. For example, incorporating greater sectoral detail would allow us to better understand the implications of development patterns for material demand, an important question as large countries develop. Directly modeling externalities and environmental costs from material extraction and waste would allow for normative analysis absent in this paper, but highly relevant for policy makers. We believe this paper contributes towards expanding the macro modeling of physical material flows to a broader set of questions.

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Online Appendix

A.1 Data Construction and Additional Figures

The first step of our data construction is to measure materials inputs, outputs (waste) and recycling rates, plus associated prices. To measure aggregate material inflows we use the United Nations (International Resource Panel) Global Material Flows Accounts (MFA). For data on waste generation (material outflow) and recycling, we rely on U.S. Environmental Protection Agency (EPA) data on municipal solid waste (MSW).[24] Prices are expressed in real terms (2018 dollars).

Four different aggregate measures of virgin materials are constructed to capture various aspects of materials usage. These measures range from including all materials and energy sources to excluding specific categories like non-metallic minerals for construction. Our benchmark measure of virgin materials quantity (TPM_EEC) is the Total Products Materials (TPM) Excluding Energy and Non Metallic Minerals - Construction Dominant (EEC), which includes ferrous and non ferrous ores, non metallic minerals - industrial or agricultural dominant, and wood. It does not include crops residues, crops, grazed biomass, and wild catch and harvest. We exclude construction-dominant materials in our benchmark measure as industrial and construction waste are not capture in the EPA data. Recycled materials' quantity is determined by summing the recycling and composting quantities in thousands of tons for various categories such as paper, glass, metals, plastic, and all other materials; composting is excluded.

To construct virgin materials prices, we collect data on the proportions of various materials within the Total Products Materials, normalizing this data to 2018 prices and calculating weighted prices. Recycled materials' prices are derived from historical data, with the assumption that the price trends for recycled commodities are similar to those of virgin commodities. The process involves resetting base years, using 2018 levels of recycled commodity prices, and constructing price paths for individual materials categories. The primary price series used for recycled products is based on products generated rather than recycled, aligning with the understanding that resource recovery is driven by prices.

The recycling rate is calculated by considering data on materials generation and recycling rates for paper, glass, metals, plastic, and all other (rubber, leather, textiles, wood. Electrolytes in lead acid batteries are also included but may not be recycled). We do not include food and yard trimmings in the measured recycling rate. The economy's overall recycling

[24] For details, see MFA: https://www.resourcepanel.org/global-material-flows-database; U.S. EPA: https://www.epa.gov/facts-and-figures-about-materials-waste-and-recycling.

rate is determined as a weighted average of these individual material recycling rates.[25]

A.1.1 Virgin, Recycled Materials and Recycling Rate

Virgin Materials - Quantity (Q_V): We construct four alternative measures of aggregate virgin materials to capture various aspects of materials usage. These measures range from including all materials, including energy sources, to excluding specific categories like non-metallic minerals for construction.

- **Materials (TPM_AM).** Total Products Materials (TPM) - All Materials (AM) includes: Coal (B), Ferrous Ores (E), Natural Gas (G), Non Ferrous Ores (H), Non Metallic Minerals - Construction Dominant (I), Non Metallic Minerals - Industrial or Agricultural Dominant (J), Oil Shale and Tar Sands (K), Petroleum (L), and Wood (N). It does not include Crops Residues (C), Crops (D), Grazed Biomass (F), and Wild Catch and Harvest (M). Data cover 1970 - 2017.

- **Materials (TPM_EE).** Total Products Materials (TPM) Excluding Energy (EE): $TPM_{EE} = TPM_{AM} - (B + G + K + L)$. Data cover 1970 - 2017.

- **Materials (TPM_EEC).**: Total Products Materials (TPM) Excluding Energy and Non Metallic Minerals - Construction Dominant (EEC): $TPM_{EEC} = TPM_{EE} - (I)$. Data cover 1970 - 2017.

- **Materials (TPM_EENM).**: Total Products Materials (TPM) Excluding Energy and Non Metallic $TPM_{EENM} = TPM_{EE} - (I + J)$. Data cover 1970 - 2017.

Our benchmark measure of virgin materials quantity is the Total Products Materials (TPM) Excluding Energy and Non Metallic Minerals - Construction Dominant (EEC):

Recycled Materials - Quantity (Q_R): Recycled materials' quantity is determined by summing the recycling and composting quantities in thousand tons for various categories such as paper, glass, metals, plastic, and all other materials.

$$Q_R = \sum_i \text{Recycling and Composting (1000 tons)}_i$$

for $i = \{Paper,\ Glass,\ Metals,\ Plastic,\ and\ All\ Other\}$, composting is excluded from the calculations for accuracy. Note that "Recycling and Composting (1000 tons)" is the name of the variable, but we exclude composting in our calculations.

[25] Data on materials Generation and Recycling and Composting are sourced from the US EPA Advancing Sustainable Material Management 2018 Tables and Figures, December 2020. Supporting Information.

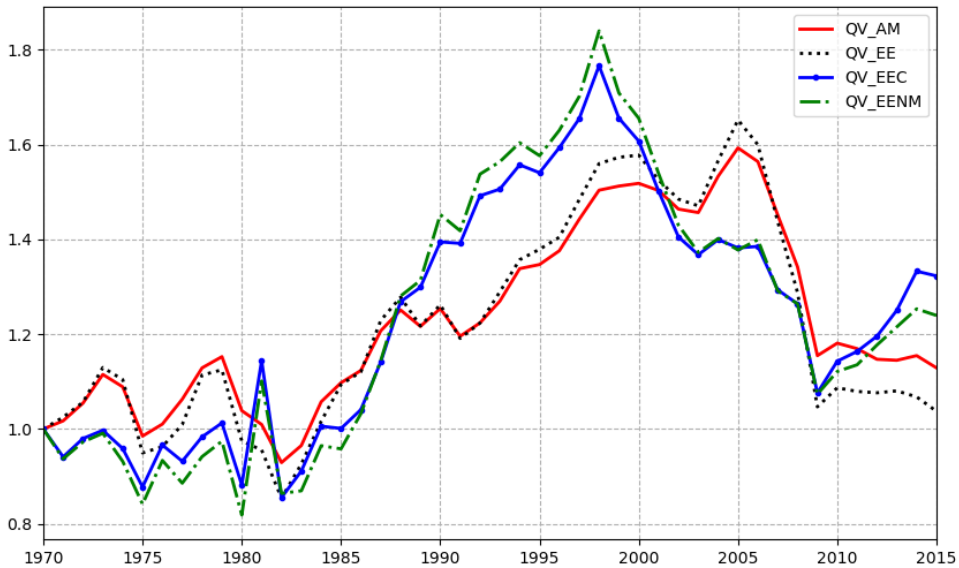

Figure A.1: Alternative Virgin Material Measures

Virgin Materials - Prices - (P_V): To construct virgin materials prices, we collect data on the proportions of various materials within the Total Products Materials and material prices (2018 Commodity Data Price per Tonne). The variable Virgin Materials - Prices (P_V) is constructed as follows:

$$P_V \equiv P_V^{TPM_{AM}} = \sum_i P_{V,i} \left(\frac{TPM_i}{TPM_{AM}} \right)$$

Real virgin materials prices (P_V^{Real}) are defined as

$$P_V^{Real} = P_V \times GDP_{Deflator},$$

where $GDP_{Deflator}$ = FRED GDPDEF (Implicit Price Deflator 2018 = 100).

We construct real virgin materials price for three other alternatives:

- Prices Virgin Materials - Total Products Materials (TPM) Excluding Energy $P_V^{TPM_{EE}}$ for $i = \{Ferrous(E),\ NonFerrous(H),\ NonMetallic(I, J),\ Wood(N)\}$
- Prices Virgin Materials - Total Products Materials (TPM) Excluding Energy and Non Metallic Minerals - Construction Dominant $P_V^{TPM_{EEC}}$ for $i = \{Ferrous(E),\ NonFerrous(H),\ NonMetallic(J),\ Wood(N)\}$

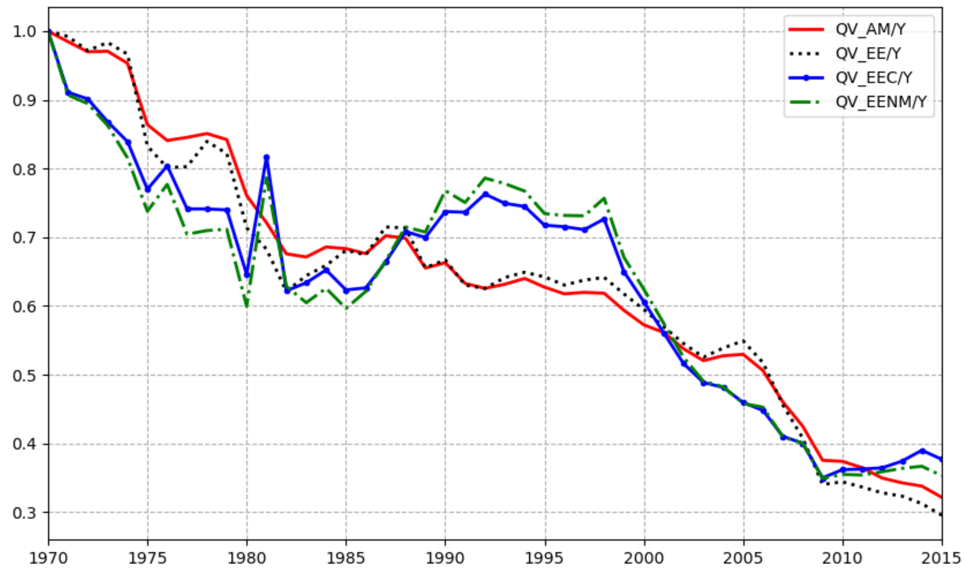

Figure A.2: Alternative Virgin Material Measures - Share GDP

- Prices Virgin Materials - Total Products Materials (TPM) Excluding Energy and Non Metallic Minerals
  $P_V^{TPM_{EENM}}$ for $i = \{Ferrous(E),\ NonFerrous(H),\ Wood(N)\}$

Recycled Materials - Prices (P_R): Recycled materials' prices are derived from historical data. The underlying assumption to construct this variable is that the path (but not level) of recycled commodity prices is similar to that of virgin commodity prices. First, we collect historical data for representative price indices for the detailed product categories - Source: Federal Reserve Economic Data - Variables Paper (WPU09), Glass (WPU131), Ferrous (WPU1012), Aluminum (WPU102501), Metals (WPU10), Rubber-Plastics (WPU07), Textiles (WPU03), and Wood (WPU08). We use PPI indices (PPI by commodity; base 1992) as we are unable to find historical data on recycled commodity prices prior to 1990. Then, we construct a set of product price series - using the historical price indices, we reset the base years to 1990 and use reported 2018 levels of recycled commodity prices to construct individual price paths. Finally, as described below we report the weighted price index for generated MSW. This is the primary price series used for recycled products. Note that because resource recovery is driven by prices, we construct and use the price index based on products generated rather than recycled.

We use the variable "Price (by Generation)" as this variable represents "Price Series for MSW Generated (Recovered Materials)". An alternative variable is"Price (by Recycling)".

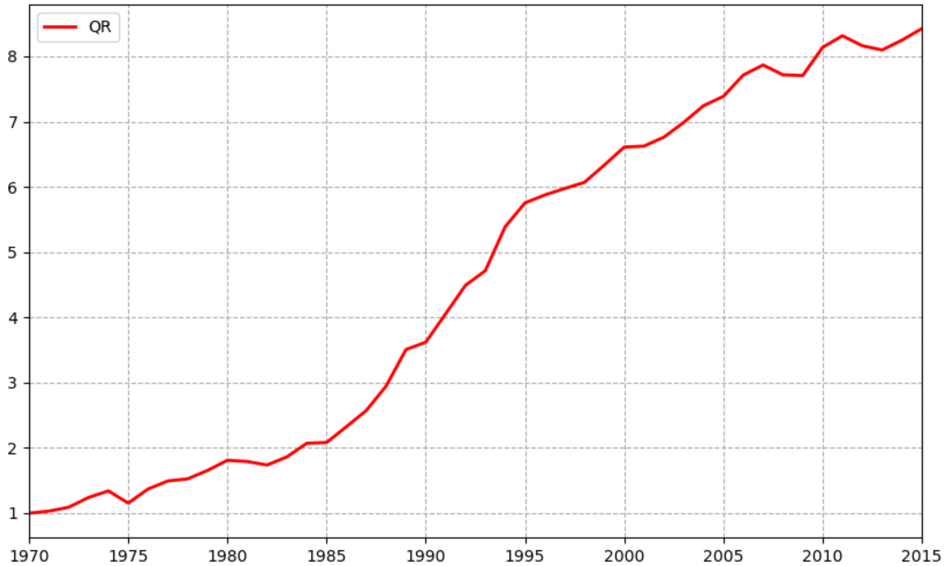

Figure A.3: Quantity Recycled (QR)

Prices (by recycling) are higher than prices (by generation) but both measures have the same overall upward trends. The variable Recycled Materials - Prices (P_R) is constructed as follows:

$$P_R = \sum_i P_i \left(\frac{RG_i}{RG_{Total}} \right)$$

for $i = \{Paper,\ Glass,\ Ferrous,\ Aluminum,\ Metals,\ Rubber/Plastic\}$, where P_i = (2018 Recycled Commodity i Prices)$\times$(Commodity i - Collected Recycled Prices), 2018 Recycled Commodity i Prices - Source: Historical Recycling Prices.

Real recycled materials prices are defined as $P_R^{Real} = P_R \times GDP_{Deflator}$

Recycling rate (RR): In order to calculate the economy's recycling rate (RR), we rely on the US EPA Advancing Sustainable Material Management 2018 Tables and Figures, December 2020. Supporting Information (https://www.epa.gov/facts-and-figures-about-materials-waste-and-recycling/studies-summary-tables-and-data-related). We consider the following materials: Paper, Glass, Metals, Plastic, and All Other (Rubber, Leather, Textiles and Wood. Electrolytes in lead acid batteries are also included but may not be recycled). We do not include Food, Yard Trimmings. Data on materials Generation (1000 tons) and Recycling and Composting (1000 tons) are sourced from the US EPA Advancing Sustainable Material Management 2018 Tables and Figures, December 2020. Supporting Information

For each material $i = \{Paper,\ Glass,\ Metals,\ Plastic,\ and\ All\ Other\}$, we collect data

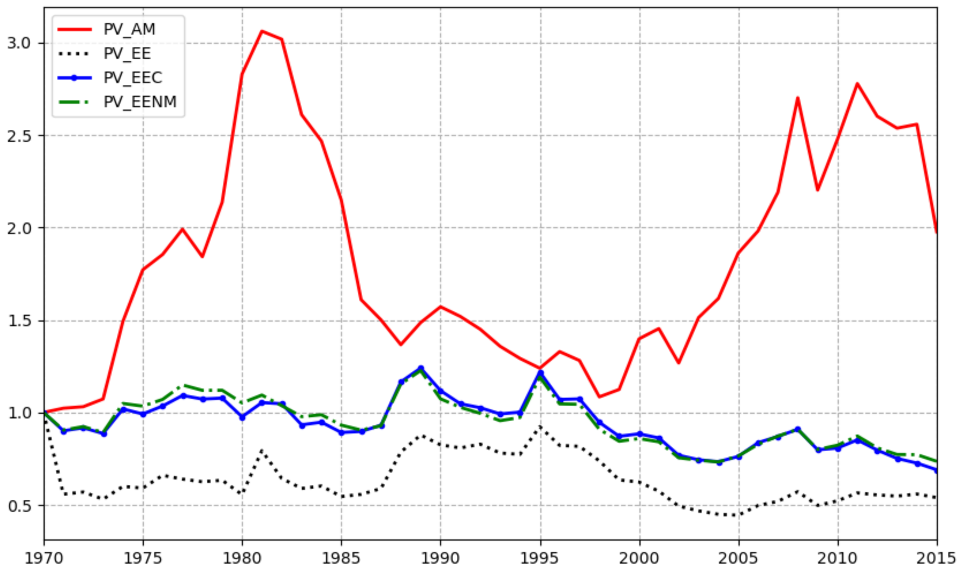

Figure A.4: Alternative Virgin Material Price Measures

on its generation G_i and calculate a material i recycling rate (RR_i) as

$$RR_i = \frac{\text{Recycling and Composting (1000 tons)}_i}{\text{Generation (1000 tons)}_i}$$

and $G_{Total} = \sum_i G_i$. The economy's recycling rate (RR) is hence a weighted average of materials i recycling rate and it is defined as follows

$$RR = \sum_i \left(\frac{G_i}{G_{Total}} \right) RR_i.$$

Figure A.6 - Recycling Rate and EPA 2030 Goal. On November 17, 2020 at the America Recycles Summit, the EPA Administrator announced the National Recycling Goal to increase the U.S. recycling rate to 50 percent by 2030. (`https://www.epa.gov/recyclingstrategy/us-national-recycling-goal`).

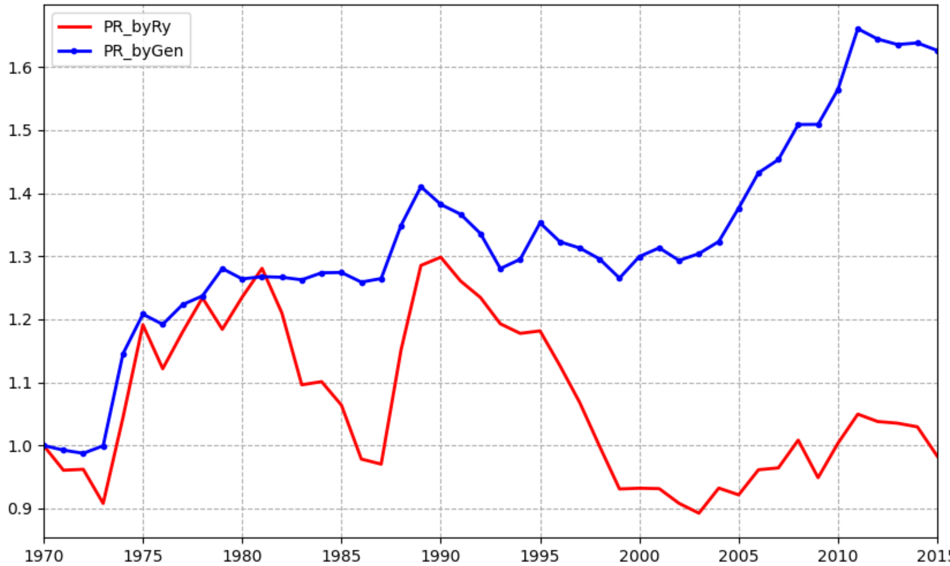

Figure A.5: Recycled Material Prices
Price by Recycling; Price by Generation

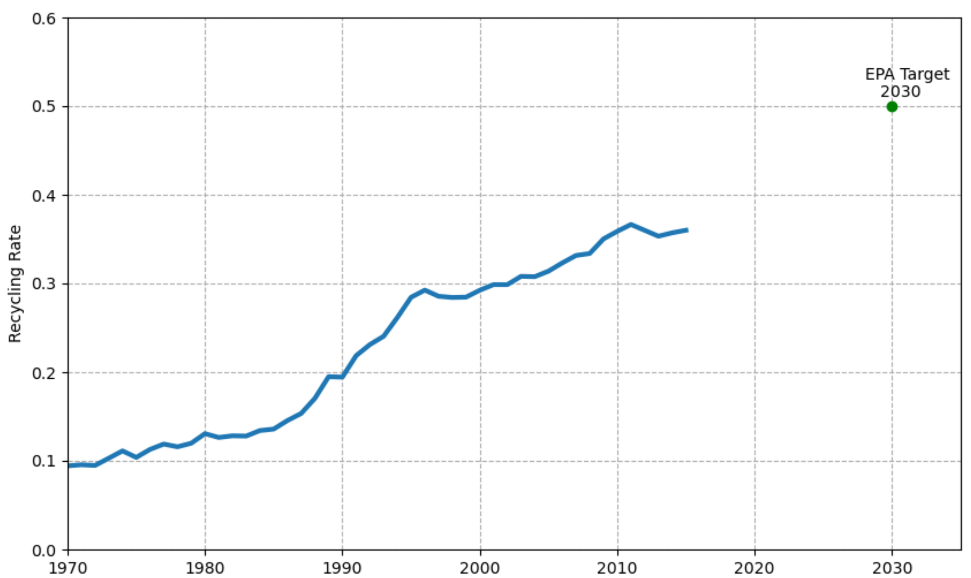

Figure A.6: Recycling Rate and EPA 2030 goal

A.2 A Growth Model of Material Use And Recycling

A.2.1 Representative Household

The representative household solves the following utility maximization problem:

$$\max_{X_t, Q_t, K_{t+1}} \sum_{0}^{\infty} \beta^t u(Q_t)$$

$$Q_t + X_t \leq w_t L_t + i_t K_t + p_{d,t} D + \pi_t^y + \pi_t^v + \pi_t^r - T_t \tag{A.1}$$

$$K_{t+1} = (1 - \delta_K) K_t + X_t \tag{A.2}$$

$$L_{t+1} = g_n L_t \tag{A.3}$$

First-order conditions with respect to X_t, Q_t, K_{t+1} yield the following euler equation:

$$u'(Q_t) = \beta u'(Q_{t+1})(i_{t+1} + 1 - \delta_K) \tag{A.4}$$

A.2.2 Virgin Primary Materials Producers

The producer, taking $\bar{V}_t$ as given, solves

$$\pi_{v,t} = \max_{V_t} V_t \left(p_{v,t} - A_{v,t} \bar{V}_t^{\psi} \right)$$

Thus quantities are determined by demand and the price must equal marginal cost, so that

$$p_{v,t} = A_{v,t} \bar{V}_t^{\psi} \tag{A.5}$$

A.2.3 Recycled Primary Materials Producers

The recycled primary materials producer solves

$$\pi_{r,t} = \max_{s_t, d_{rt}} p_{r,t} A_{r,t} \left(W_t s_t \right)^{\alpha_r} - \hat{c}_t \left(\frac{W_t}{\bar{\gamma}_t} \right) s_t - p_{d,t} d_{r,t} \tag{A.6}$$

$$A_{r,t} \left(W_t s_t \right)^{\alpha_r} \leq W_t$$

$$A_{r,t} = g_{A_R}(d_{rt}) A_{r,t-1}$$

The recycler chooses effort (processing) s_t to increase recyclability, at proportional cost $\hat{c}_t$. It recovers quantity of materials R_t that may not exceed quantity collected and invests in improvements in recycling technology. Notice that the recycling cost $\hat{c}_t = c_t - \tau_t$ may be subsidized. The solution of the recycler's problem implies the following representation of recycled materials:

$$R_t = \left(\frac{p_{rt}\alpha_r}{\hat{c}_t}\right)^{\frac{\alpha_r}{1-\alpha_r}} A_r^{\frac{1}{1-\alpha_r}} \bar{\gamma}_t^{\frac{\alpha_r}{1-\alpha_r}} \tag{A.7}$$

A.2.4 Final Goods Producer

We assume that $A_{n,t} = g_{A_n} A_{n,t-1}$ and $A_{m,t} = g_{A_m} A_{m,t-1}$ are resource-saving technologies for the capital-labor composite and the material composite, respectively, where $g_{A_j} = 1 + \eta_j d_{j,t}^{1-\lambda}$, for $j = \{n, m\}$. The final goods producer solves the following profit maximization problem

$$\max_{K_t, L_t, V_t, R_t, d_{n,t}, d_{m,t}} \quad Y_t - w_t L_t - i_t K_t - (1+\tau_v) p_{v,t} V_t - p_{r,t} R_t - p_{d,t}(d_{n,t} + d_{m,t}). \tag{A.8}$$

where

$$Y_t = \left(\delta_y (A_{n,t} K_t^{\alpha} L_t^{1-\alpha})^{\frac{\epsilon-1}{\epsilon}} + (1-\delta_y)(A_{m,t}\tilde{M}_t)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}},$$

and

$$\tilde{M}_t = \left[(1-\delta_R) V_t^{\frac{\epsilon_m - 1}{\epsilon_m}} + \delta_R R_t^{\frac{\epsilon_m - 1}{\epsilon_m}}\right]^{\frac{\epsilon_m}{\epsilon_m - 1}}$$

The first-order conditions with respect to $K_t, L_t, V_t, R_t, d_{n,t}, d_{m,t}$ are as follows, respectively

$$i_t = Y_t^{\frac{1}{\epsilon}} \delta_y (A_{nt} K_t^{\alpha} L_t^{1-\alpha})^{-\frac{1}{\epsilon}} \alpha A_{nt} K_t^{\alpha-1} L_t^{1-\alpha} \tag{A.9}$$

$$w_t = Y_t^{\frac{1}{\epsilon}} \delta_y (A_{nt} K_t^{\alpha} L_t^{1-\alpha})^{\frac{-1}{\epsilon}} (1-\alpha) A_{nt} K_t^{\alpha} L_t^{-\alpha} \tag{A.10}$$

$$(1+\tau_v) p_{v,t} = Y_t^{\frac{1}{\epsilon}} (1-\delta_y)(A_{mt}\tilde{M}_t)^{\frac{\epsilon-1}{\epsilon}} (1-\delta_R) \left(\frac{V_t}{\tilde{M}_t}\right)^{\frac{\epsilon_m - 1}{\epsilon_m}} \tag{A.11}$$

$$p_{r,t} = Y_t^{\frac{1}{\epsilon}} (1-\delta_y)(A_{mt}\tilde{M}_t)^{\frac{\epsilon-1}{\epsilon}} (\delta_R) \left(\frac{R_t}{\tilde{M}_t}\right)^{\frac{\epsilon_m - 1}{\epsilon_m}} \tag{A.12}$$

$$p_{d,t} = Y_t^{\frac{1}{\epsilon}} \delta_y (A_{nt} K_t^{\alpha} L_t^{1-\alpha})^{\frac{\epsilon-1}{\epsilon}} \frac{(1-\lambda)\eta_n d_{n,t}^{-\lambda}}{1 + \eta_n d_{n,t}^{1-\lambda}} \tag{A.13}$$

$$p_{d,t} = Y_t^{\frac{1}{\epsilon}} (1-\delta_y)(A_{mt}\tilde{M}_t)^{\frac{\epsilon-1}{\epsilon}} \frac{(1-\lambda)\eta_m d_{m,t}^{-\lambda}}{1 + \eta_m d_{m,t}^{1-\lambda}} \tag{A.14}$$

A.2.5 Government, Laws of Motion and Resource Constraints

The government budget constraint is as follows

$$T_t + \tau_v p_{v,t} V_t = \tau \left(\frac{W_t}{\bar{\gamma}_t}\right) s_t \tag{A.15}$$

Consumption, capital and materials laws of motion, respectively

$$C_{t+1} = (1 - \delta_C)C_t + Q_t \tag{A.16}$$

$$K_{t+1} = (1 - \delta_K)K_t + X_t \tag{A.17}$$

$$\bar{\gamma}_{t+1} = \frac{(1 - \delta_C)C_t}{C_{t+1}}\bar{\gamma}_t + \frac{Q_t}{C_{t+1}}\gamma_t \tag{A.18}$$

And, the economy resource constraint is given by

$$Y_t = Q_t + X_t + c\left(\frac{W_t}{\bar{\gamma}_t}\right)s_t + V_t A_{vt}\bar{V}_t^{\psi} \tag{A.19}$$

A.2.6 Competitive Equilibrium

Definition 1. *An equilibrium is a sequence of consumer stocks and choices* $\{C_t, Q_t, K_t, X_t\}$*, virgin extraction decisions* $\{V_t\}$*, recycler decisions* $\{S_{ut}, d_{r,t}\}$ *and final goods producer decisions* $\{K_t^f, L_t^f, V_t^f, R_t^f, d_{n,t}, d_{m,t}\}$ *along with initial conditions* $\{C_0, K_0, L_0, \bar{\gamma}_0, A_{n,0}, A_{m,0}, A_{r,0}\}$*, government policy* $\{T_t\}, \tau_v, \tau_t$ *and prices* $\{w_t, i_t, p_{d,t}, p_{v,t}, p_{r,t}\}$ *such that*

1. *Consumer optimality, equation (A.4) is satisfied.*

2. *Virgin material producer, equation (A.5), recycled material producer, equation (A.7, and final good producer, equations (A.9)-A.14), optimality conditions are satisfied.*

3. *All stocks evolve according to their laws of motion.*

$$C_{t+1} = (1 - \delta_C)C_t + Q_t$$
$$K_{t+1} = (1 - \delta_K)K_t + X_t$$
$$\bar{\gamma}_{t+1} = \frac{(1 - \delta_C)C_t}{C_{t+1}}\bar{\gamma}_t + \frac{Q_t}{C_{t+1}}\gamma_t$$

4. *The government budget constraint, equation (A.15) balances every period.*

5. *The following market clearing conditions are satisfied: Markets clear:* K, V, R, D

$$D = \sum_j d_{jt} \tag{A.20}$$

A.2.7 Single Firm Profit Maximization Problem

For convenience, we collapse the three producer problems into a single problem. The problems are identical as long as we assume producers do not internalize how their material use choices in final output impact material shares in the economy, which then feed into the recycler constraints one period ahead. Thus the equivalent problem is a single firm solving

$$\max_{\substack{K_t, L_t, S_{ut}, V_t, \\ \{d_{j,t}\}_{j\in\{n,m,r\}}}} \quad Y - w_t L_r - i_t K_t - (1+\tau_v) A_{vt} \bar{V}_t^{\psi} V_t - \hat{c}\left(\frac{W_t}{\bar{\gamma}_t}\right) s_t \tag{A.21}$$

$$Y = \left(\delta_y (g_{A_n}(d_{nt}) A_{n,t-1} K_t^{\alpha} L_t^{1-\alpha})^{\frac{\epsilon-1}{\epsilon}} + (1-\delta_y)\left(g_{A_m}(d_{mt}) A_{m,t-1} \tilde{M}_t\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$$

$$\tilde{M}_t = \left[\delta_R R_t^{\frac{\epsilon_m-1}{\epsilon_m}} + (1-\delta_R) V_t^{\frac{\epsilon_m-1}{\epsilon_m}}\right]^{\frac{\epsilon_m}{\epsilon_m-1}}$$

$$R_t = g_{A_r}(d_{rt}) A_{r,t-1} (W_t s_t)^{\alpha_r}$$

$$R_t \leq W_t$$

$$\sum_j d_{jt} \leq D$$

where $g_{A_j} = 1 + \eta_j d_{j,t}^{1-\lambda}$ and $\hat{c} = c - \tau$. This yields the following FOCs

$$F_{Kt} = Y_t^{\frac{1}{\epsilon}} \delta_y (A_{nt} K_t^{\alpha} L_t^{1-\alpha})^{\frac{-1}{\epsilon}} \alpha A_{nt} K_t^{\alpha-1} L_t^{1-\alpha} = i_t$$

$$F_{Lt} = Y_t^{\frac{1}{\epsilon}} \delta_y (A_{nt} K_t^{\alpha} L_t^{1-\alpha})^{\frac{-1}{\epsilon}} (1-\alpha) A_{nt} K_t^{\alpha} L_t^{-\alpha} = w_t$$

$$F_{Vt} = Y_t^{\frac{1}{\epsilon}} (1-\delta_y)(A_{mt}\tilde{M}_t)^{\frac{\epsilon-1}{\epsilon}} (1-\delta_R)\left(\frac{V_t}{\tilde{M}_t}\right)^{\frac{\epsilon_m-1}{\epsilon_m}} = V_t (1+\tau_v) A_{vt} \bar{V}_t^{\psi}$$

$$F_{s_t} = Y_t^{\frac{1}{\epsilon}} (1-\delta_y)(A_{mt}\tilde{M}_t)^{\frac{\epsilon-1}{\epsilon}} \delta_R \alpha_r \left(\frac{R_t}{\tilde{M}_t}\right)^{\frac{\epsilon_m-1}{\epsilon_m}} = \hat{c}\left(\frac{W_t}{\bar{\gamma}_t}\right) s_t + \mu_r$$

$$F_{d_{nt}} = Y_t^{\frac{1}{\epsilon}} \delta_y (A_{nt} K_t^{\alpha} L_t^{1-\alpha})^{\frac{\epsilon-1}{\epsilon}} \frac{g'_{A_n}(d_{nt})}{g_{A_n}(d_{nt})} = \mu_{dt}$$

$$F_{d_{mt}} = Y_t^{\frac{1}{\epsilon}} (1-\delta_y)(A_{mt}\tilde{M}_t)^{\frac{\epsilon-1}{\epsilon}} \frac{g'_{A_m}(d_{mt})}{g_{A_m}(d_{mt})} = \mu_{dt}$$

$$F_{d_{rt}} = Y_t^{\frac{1}{\epsilon}} (1-\delta_y)(A_{mt}\tilde{M}_t)^{\frac{\epsilon-1}{\epsilon}} \delta_R \left(\frac{R_t}{\tilde{M}_t}\right)^{\frac{\epsilon_m-1}{\epsilon_m}} \frac{g'_{A_r}(d_{rt})}{g_{A_r}(d_{rt})} = \mu_{dt}$$

where μ_d is the multiplier on the fixed research factor and μ_r is the multiplier on the recyclability effort s_t. In the following analysis we will assume the recycler solutions are always interior, so that the constraints do not bind.

A.2.8 Characterization of Optimal Innovation - Section 3.3.2

From the first-order conditions of firm's problem presented above (Section A.2.7) we recover the following pair of conditions:

$$\delta_y(A_{nt}K_t^{\alpha}L_t^{1-\alpha})^{\frac{\epsilon-1}{\epsilon}}\frac{g'_{A_n}(d_{nt})}{g_{A_n}(d_{nt})} = (1-\delta_y)(A_{mt}\tilde{M}_t)^{\frac{\epsilon-1}{\epsilon}}\frac{g'_{A_m}(d_{mt})}{g_{A_m}(d_{mt})} \tag{A.22}$$

$$\delta_R\left(\frac{R_t}{\tilde{M}_t}\right)^{\frac{\epsilon_m-1}{\epsilon_m}}\frac{g'_{A_r}(d_{rt})}{g_{A_r}(d_{rt})} = \frac{g'_{A_m}(d_{mt})}{g_{A_m}(d_{mt})} \tag{A.23}$$

Note that the ratio g'_{A_j}/g_{A_j} is the elasticity of technology growth, and is decreasing in the research allocation d_j. Dividing both sides of Equation A.22 by $Y^{\frac{\epsilon-1}{\epsilon}}$ we can express this as:

$$\theta^Y_{KL}\frac{\eta_n d_n^{-\lambda}}{(1+\eta_n d_n^{1-\lambda})} = \theta^Y_M\frac{\eta_m d_m^{-\lambda}}{(1+\eta_m d_m^{1-\lambda})} \tag{A.24}$$

where θ^Y_{KL} is the output share of capital and labor expenditure, and θ^Y_M is the output share of material expenditure (adjusted for the profits earned on the recycling technology).

Similarly, we can express Equation A.23 as

$$\frac{\eta_m d_m^{-\lambda}}{(1+\eta_m d_m^{1-\lambda})} = \theta^M_R\frac{\eta_m d_r^{-\lambda}}{(1+\eta_r d_r^{1-\lambda})} \tag{A.25}$$

where θ^M_R is the expenditure on material recovery as a share of total material spending. While our formulation abstracts from formally modeling researchers as in Acemoglu et al. (2012), we recover the same basic mechanism - indeed, Equation 20 is nearly identical to optimal innovation in Casey (2023) (see expression (19) on p. 12).

A.2.9 Proof of Proposition 1

Proposition 1. *For static producer decisions*

1. *The material mix $\frac{R_t}{V_t}$ is increasing in scarcity $\bar{V}_t$, virgin tax and recycling subsidy.*

2. *The material share of output $\frac{\tilde{M}_t}{Y_t}$ is decreasing in scarcity $\bar{V}_0$ and the virgin tax, and increasing in the recycling subsidy.*

3. *The recycling rate is increasing in the recycling subsidy and ambiguous in scarcity, the virgin tax.*

Proof. The proposition analyzes how three different material ratios respond to scarcity (which we proxy using cumulative extraction $\bar{V}_t$), taxes and subsidies. We discuss each in turn. Combining the first order conditions for V_t and S_{ut} we obtain an expression for the optimal material mix:

$$\frac{1-\delta_R}{\delta_R}\left(\frac{R_t}{V_t}\right)^{\frac{1}{\epsilon_m}}\frac{S_{ut}}{\alpha_r R_t} = \frac{\tau_v A_{vt}\bar{V}_t^{\psi}}{\hat{c}}$$
$$\frac{1-\delta_R}{\delta_R}\left(\frac{R_t}{V_t}\right)^{\frac{1}{\epsilon_m}}\frac{1}{\partial R/\partial S_{ut}} = \frac{\tau_v A_{vt}\bar{V}_t^{\psi}}{\hat{c}}$$

As virgin material becomes relatively more expensive (through greater extraction, higher taxes or lower recovery costs net of subsidy) the right-hand side goes up. For the left-hand side to increase, S_{ut} must become relatively larger - increasing the ratio of recycled to virgin material and lowering the marginal return to collected units S_{ut} given curvature in the production technology. This yields part one of the proposition.

Turning to the material share of output, we can rearrange the optimality condition for virgin material to obtain:

$$\left(\frac{Y_t}{\tilde{M}_t}\right)^{\frac{1}{\epsilon}}(1-\delta_y)(A_{mt})^{\frac{\epsilon-1}{\epsilon}}(1-\delta_R)\left(\frac{V_t}{\tilde{M}_t}\right)^{\frac{1}{\epsilon_m}} = p_{vt} \tag{A.26}$$

This equation relates the price of virgin materials to the inverse of the composite material share of output and the virgin share of the composite. With some straightfoward manipulation of the materials composite, one can also show that

$$\frac{V_t}{\tilde{M}_t} = \frac{1}{\left[\delta_R\left(\frac{R_t}{V_t}\right)^{\frac{\epsilon_m-1}{\epsilon_m}}+(1-\delta_R)\right]^{\frac{\epsilon_m}{\epsilon_m-1}}} \tag{A.27}$$

The expression for optimal material mix already established that as p_{vt} rises, $\frac{R_t}{V_t}$ rises, which above we can see implies $\frac{V_t}{\tilde{M}_t}$ falls. Since the virgin share of the material composite falls, this implies the ratio $\frac{\tilde{M}_t}{Y_t}$ must fall. This is part two of the proposition.

Finally, in static terms the recycling rate is entirely determined by the recycling quantity R_t, as we take the waste quantity W_t as given. Solving for s_t from the FOC and plugging this into the production function for recycling, we obtain

$$R_t^{1-\alpha_r+\frac{\alpha_r}{\epsilon_m}} = A_{rt}\left((1+\tau_{vt})P_{vt}V_t^{\frac{1}{\epsilon_m}}\frac{\delta_R\alpha_r}{1-\delta_R}\frac{\bar{\gamma}_t}{\hat{c}}\right)^{\alpha_r} \tag{A.28}$$

The direct effect of higher prices through taxes or scarcity, or a decline in the cost $\hat{c}$ through a rising subsidy, is to increase recycling. However, there is a potentially offsetting effect captured by the V_t term. When virgin prices rise either through taxes or scarcity, the incentive to recycle rises while the overall demand for materials declines, lowering V_t. This offsetting effect makes the impact on the recycling rate ambiguous. In the case of subsidies however, the overall cost of materials falls, so V_t rises as well, and recycling (and the recycling rate) rise. This is part 3 of the proposition.

■

A.2.10 Proof of Proposition 2

Proposition 2. *For any set of government policies τ, τ_v, labor growth rate g_n and growth rate of virgin extraction TFP g_{Av}, if a BGP exists then it is unique and:*

1. *Y_t, C_t, K_t, S_{ut} grow at rate $g_y = g_n g_{A_n}^{\frac{1}{1-\alpha}}$*
2. *$M_t, V_t, R_t, \bar{V}_t$ grow at rate $\left(\frac{g_y}{g_{A_v}}\right)^{\frac{1}{1+\psi}}$*
3. *$\gamma_t, \bar{\gamma}_t$ grow at rate $\left(g_y^{\psi} g_{A_v}\right)^{\frac{-1}{1+\psi}}$*
4. *Technology growth rates (and R&D allocations) are determined by the unique solution to the system of equations:*

$$g_{Am}(d_m) = g_{An}(d_n)^{\frac{\psi}{(1-\alpha)(1+\psi)}} g_n^{\frac{\psi}{1+\psi}} g_{Av}^{\frac{1}{1+\psi}}$$
$$g_{Ar}(d_r) = g_{An}(d_n)^{\frac{1}{1+\psi}} \left(\frac{g_n}{g_{Av}}\right)^{\frac{1-\alpha_r}{1+\psi}}$$
$$1 = d_r + d_m + d_n$$

Proof. We consider a Balanced Growth Path (BGP) where C, K, Y grow at constant identical rates, all other aggregates and technologies grow at constant rates, i_t is constant, as are income shares. We also assume $c, \tau, \hat{c}$ are constant. To analyze the GBP we will assume $\hat{c} = c - \tau$ is constant. We have the following sets of variables to characterize:

$$Y_t, L_t, S_{ut}, R_t, V_t, \{d_{jt}\}, C_t, K_t, X_t, Q_t, T_t \quad \textbf{Choices}$$
$$A_{vt}, \{A_{jt}\} \quad \textbf{TFPs}$$
$$\bar{\gamma}_t, \gamma_t, \bar{V}_t \quad \textbf{Other LoM}$$

We start our analysis by considering what restrictions are imposed by the resource constraints and laws of motion. Consider first the LOM for consumption. Since C_t, Q_t grow at constant rates, it must be that they grow at the identical rate, so $g_c = g_q = g_y$ (the last is by imposition of this characteristic on our BGP). The same argument using the capital law of motion implies $g_k = g_x = g_y$.

Turning to the resource constraint, if all elements are growing at a constant rate, then the same argument implies they grow at the same rate, so that

$$\begin{aligned} g_{S_u} &= g_y \\ g_v g_{A_v} g_{\bar{V}}^{\psi} &= g_y \end{aligned} \tag{A.29}$$

Next, use the production technology for recycling and the imposition of constant growth in R_t to write

$$g_r = g_{A_r} \left(g_{\bar{\gamma}} g_{S_u} \right)^{\alpha_r} = g_{A_r} \left(g_{\bar{\gamma}} g_y \right)^{\alpha_r} \tag{A.30}$$

where the constant growth of $\bar{\gamma}$ is implied by the equation above, as all other elements are constants.

Turning to the LOM for $\bar{\gamma}$ we rewrite this as:

$$g_{\bar{\gamma}} = \frac{1 - \delta_c}{g_c} + \frac{q_t}{C_t} \frac{\gamma_t}{\bar{\gamma}_t} \tag{A.31}$$

However, we know that q_t/C_t is constant (they grow at a constant rate) thus $\frac{\gamma_t}{\bar{\gamma}_t}$ is constant, implying $g_{\bar{\gamma}} = g_{\gamma} = \frac{g_m}{g_y}$ where the last equality comes from the definition of $\gamma_t = \frac{\rho M_t}{Y_t}$.

The last LOM is for cumulative extraction. Assuming this also grows at a constant rate we obtain:

$$g_{\bar{V}} = 1 + \frac{V_t}{\bar{V}_{t+1}} \tag{A.32}$$

Since the LHS is constant, the RHS ratio is as well, which implies that $g_v = g_{\bar{V}}$. Plugging this back into the expression A.29 we obtain

$$g_v^{1+\psi} g_{A_v} = g_y \tag{A.33}$$

To compute the growth rate of materials, note:

$$\frac{M_{t+1}}{M_t} = \frac{g_{A_m}(V_t g_v + \delta_m R_t g_r)}{V_t + \delta_m R_t} \tag{A.34}$$

Again, constant growth rates in all materials imply that these growth rates must be the same, so that $g_m = g_r = g_v$. We can then derive:

$$
\begin{aligned}
g_m &= g_r \\
g_m &= g_{A_r} \left(g_{\bar{\gamma}} g_y\right)^{\alpha_r} \\
g_m &= g_{A_r} \left(g_m\right)^{\alpha_r} \\
g_m &= g_{A_r}^{\frac{1}{1-\alpha_r}}
\end{aligned}
\tag{A.35}
$$

The first line uses the fact that all material aggregates grow at a constant rate. The second substitutes in for g_r. The third substitutes in for $g_{\bar{\gamma}}$ to get the cancellation and the last line isolates g_m.

We can then turn to the growth rate of virgin materials:

$$
\begin{aligned}
g_v^{1+\psi} g_{A_v} &= g_y \\
g_{A_r}^{\frac{1+\psi}{1-\alpha_r}} g_{A_v} &= g_y \\
g_{A_r} &= \left(\frac{g_y}{g_{A_v}}\right)^{\frac{1-\alpha_r}{1+\psi}}
\end{aligned}
\tag{A.36}
$$

where the first line is the expression for the growth rate, the second line plugs in $g_m = g_v$ using the expression for materials growth in the steps above. The last line isolates the growth rate of A_r in terms of g_y and g_{A_v}.

Plugging this expression back into the one above we obtain:

$$
g_m = g_v = g_r = \left(\frac{g_y}{g_{A_v}}\right)^{\frac{1}{1+\psi}} \tag{A.37}
$$

Further, we know

$$
g_\gamma = g_{\bar{\gamma}} = \frac{g_m}{g_y} = \left(g_y^{\psi} g_{A_v}\right)^{\frac{-1}{1+\psi}} \tag{A.38}
$$

Finally, we have the lump sum transfer. Since $T_t = \tau S_{ut}$ we have immediately that $g_T = g_{S_u} = g_y$.

Everything is now written in terms of one unknown (g_y) and parameters (recall that g_{A_v} is exogenous). At this point it is useful to pause and summarize what we know about growth rates of different variables:

$$\begin{aligned} L_t &: g_n \\ A_{vt} &: g_{A_v} \\ S_{ut}, Y_t, C_t, K_t, X_t, Q_t, T_t &: g_y \\ M_t, V_t, R_t, \bar{V}_t &: \left(\frac{g_y}{g_{A_v}}\right)^{\frac{1}{1+\psi}} \\ \gamma_t, \bar{\gamma}_t &: \left(g_y^{\psi} g_{A_v}\right)^{\frac{-1}{1+\psi}} \end{aligned}$$

The only variables remaining are the growth rates of technology A_{mt}, A_{nt}, A_{rt} which are determined by allocations of the fixed resource $\{d_{mt}, d_{nt}, d_{rt}\}$, in addition to the unknown g_y. We next turn to the optimality conditions and derive their implications for technology growth. First, note that technological growth rates are determined by the fixed resource allocations, so constant growth implies constant allocations.

Setting $F_{d_{mt}} = F_{d_{rt}}$ yields the requirement that $\frac{M_t}{\delta_m R_t}$ is constant, which implies identical growth rates of materials (this was already derived above).

Setting $F_{d_{mt}} = F_{d_{nt}}$ implies

$$\frac{A_{nt} K_t^{\alpha} L_t^{1-\alpha}}{A_{mt}(V_t + \delta_m R_t)} \tag{A.39}$$

is a constant. Thus we obtain

$$\frac{g_{A_n} g_k^{\alpha} g_n^{1-\alpha}}{g_{A_m} g_m} = 1 \tag{A.40}$$

A constant capital share of income implies we can rewrite the FOC for capital to obtain the following expression

$$\alpha \delta_y \frac{A_{nt} K_t^{\alpha} L_t^{1-\alpha}}{Y_t} = \frac{i K_t}{Y_t} \tag{A.41}$$

where i is the constant interest rate on the EBGP. This expression then implies

$$\frac{g_{A_n} g_k^{\alpha} g_n^{1-\alpha}}{g_y} = 1 \Rightarrow g_y = g_{A_n}^{\frac{1}{1-\alpha}} g_n \tag{A.42}$$

The same approach for materials expenditure re-expresses income share as:

$$(1-\delta_y) \frac{A_{mt} M_t}{Y_t} = \frac{p_{mt} M_t}{Y_t} \tag{A.43}$$

The LHS being constant then yields:

$$\frac{g_{A_m} g_m}{g_y} = 1 \Rightarrow g_y = g_{A_m} g_m = g_{A_n}^{\frac{1}{1-\alpha}} g_n \tag{A.44}$$

If we return to the expression for the interest rate, written in terms of income share, we obtain

$$\delta_y \left(\frac{A_{nt} K_t^{\alpha} L_t^{1-\alpha}}{Y_t} \right)^{\frac{1}{\epsilon}} \alpha A_{nt} \left(\frac{L_t}{K_t} \right)^{1-\alpha} = i \tag{A.45}$$

Since the large bracket ratio is constant given constant expenditure shares, this implies that $A_{nt} \left(\frac{L_t}{K_t} \right)^{1-\alpha}$ must also be constant. This then implies

$$g_{A_n} \left(\frac{g_n}{g_k} \right)^{1-\alpha} = 1 \Rightarrow g_k = g_{A_n}^{\frac{1}{1-\alpha}} = g_y \tag{A.46}$$

Finally, the FOC for S_{ut} (simply dividing by periods t and $t+1$) requires:

$$g_{A_v} g_v^{\psi} g_{A_r} g_{\tilde{\gamma}}^{\alpha_r} g_{Su}^{\alpha_r - 1} = 1 \tag{A.47}$$

Collecting the relevant equations from optimality:

$$g_y = g_{A_n}^{\frac{1}{1-\alpha}} g_n \tag{A.48}$$

$$g_y = g_{A_m} g_m \tag{A.49}$$

$$g_{A_v} g_v^{\psi} g_{A_r} g_{\tilde{\gamma}}^{\alpha_r} g_{Su}^{\alpha_r - 1} = 1 \tag{A.50}$$

Taking Equation A.49, the first line below replaces g_m in terms of output. The second line isolates for g_{Am}. The third line substitutes for g_y using Equation A.48.:

$$\begin{aligned} g_{Am} g_y^{\frac{1}{1+\psi}} &= g_y g_{Av}^{\frac{1}{1+\psi}} \\ g_{Am} &= g_y^{\frac{\psi}{1+\psi}} g_{Av}^{\frac{1}{1+\psi}} \\ g_{Am} = g_{An}^{\frac{\psi}{(1-\alpha)(1+\psi)}} & g_n^{\frac{\psi}{1+\psi}} g_{Av}^{\frac{1}{1+\psi}} \end{aligned} \tag{A.51}$$

Next take Equation A.50 and substitute in for the appropriate growth rates (line one). Line 2 isolates for g_{Ar} and line 3 plugs in for the expression of output growth in terms of

g_{An}:

$$g_{Av}\left(\frac{g_y}{g_{Av}}\right)^{\frac{\psi}{1+\psi}} g_{Ar}\left(g_y^{\psi} g_{Av}\right)^{\frac{-\alpha_r}{1+\psi}} g_y^{\alpha_r-1} = 1$$

$$g_{Ar} = \left(\frac{g_y}{g_{Av}}\right)^{\frac{1-\alpha_r}{1+\psi}}$$

$$g_{Ar} = g_{An}^{\frac{1}{1+\psi}}\left(\frac{g_n}{g_{Av}}\right)^{\frac{1-\alpha_r}{1+\psi}} \tag{A.52}$$

Equations A.51 and A.52 express the values of growth rates (hence d_m, d_r) in terms of the KL composite technological growth (d_n). Given functional forms for technological growth then, these two equations implicitly yield $d_m(d_n), d_r(d_n)$. Combining these with the fixed factor constraint $\sum d_j = D$ we have one equation with one unknown, hence have a solution for the BGP. Notice that once we have a solution for fixed factor allocations, we can compute g_y which in turn yields the constant interest rate through the Hotelling equation derived in the household problem. This completes the derivation of growth rates and equilibrium technology in the proposition. ■

A.2.11 Proof of Proposition 4

Proposition 4. *On the BGP*

1. *The material mix $\frac{R_t}{V_t}$ and material share of output $\frac{\tilde{M}_t}{Y_t}$ are independent of policies and initial scarcity $\bar{V}_0$.*

2. *The recycling rate is increasing in the recovery subsidy τ, decreasing in the virgin extraction tax τ_v, and independent of initial scarcity $\bar{V}_0$.*

Proof. As discussed in the text, part one of the proposition is a direct result from the characterization of the BGP. We focus here on part 2. The recycling rate is expressed as

$$rr_t = \frac{A_{rt}\left(\bar{\gamma}_{t-1} S_{ut}\right)^{\alpha_r}}{\bar{\gamma}_{t-1}\delta_c C_{t-1}} \tag{A.53}$$

Dividing $t+1$ into t and doing the appropriate substitutions, we obtain that rr is constant on the BGP. To characterize the value of the recycling rate on the BGP we do the following.

Start with the LOM for $\bar{\gamma}_t$:

$$\bar{\gamma}_t C_t = \bar{\gamma}_{t-1}(1-\delta_c)C_{t-1} + q_t \frac{\rho M_t}{Y_t}$$
$$\frac{\bar{\gamma}_t C_t}{\bar{\gamma}_{t-1}C_{t-1}} = (1-\delta_c) + \frac{q_t}{Y_t}\frac{\rho M_t}{\bar{\gamma}_{t-1}C_{t-1}}$$
$$g_{\bar{\gamma}} g_y = 1-\delta_c + \tilde{q}\frac{\rho M_t}{\bar{\gamma}_{t-1}C_{t-1}}$$

The last line recognizes that since q, Y grow at identical rates, the ratio is a constant. Further, we know that since all material aggregates grow at the same rate, $R_t = \frac{(1-\tilde{v})}{\delta_m}M_t$, we can rewrite the recycling rate as

$$rr = \frac{1-\tilde{v}}{\delta_m \delta_c}\frac{M_t}{\bar{\gamma}_{t-1}C_{t-1}} = \frac{1-\tilde{v}}{\delta_m \delta_c}\frac{g_{\bar{\gamma}} g_y - 1 + \delta_c}{\rho \tilde{q}} \tag{A.54}$$

Optimality of fixed factor allocations to materials and recycling technology implies

$$\frac{\delta_m R_t}{M_t} = 1-\tilde{v} = \frac{g_{Ar}}{g_{Am}}\frac{g'_{Am}}{g'_{Ar}} \tag{A.55}$$

This term, we know, is independent of policy. So the impact of policy, if any, on the BGP recycling rate will come through the term $\tilde{q}$. We obtain $\tilde{q}$ as a residual from the resource constraint:

$$1 = \tilde{q} + \tilde{x} + \frac{V_t A_{vt}\bar{V}_{t-1}^{\psi}}{Y_t} + \frac{cS_{ut}}{Y_t} \tag{A.56}$$

- $\tilde{x}$: Using optimality of d_m, d_n we can rewrite output as follows.

$$Y_t = \left[\delta_y + \delta_y \frac{g_{Am}}{g_{An}}\frac{g'_{An}}{g'_{Am}}\right]^{\frac{\epsilon}{\epsilon-1}} A_{nt}K_t^{\alpha}L_t^{1-\alpha} \tag{A.57}$$

Plugging this into the expression for MPK we obtain

$$A_{nt}K_t^{\alpha}L_t^{1-\alpha} = \frac{iK_t}{\alpha\delta_y}\left[\delta_y + \delta_y \frac{g_{Am}}{g_{An}}\frac{g'_{An}}{g'_{Am}}\right]^{\frac{-1}{\epsilon-1}} \tag{A.58}$$

Substitution back into the Y expression and rearranging yields

$$\frac{K_t}{Y_t} = \frac{\alpha}{i\left[1+\frac{g_{Am}}{g_{An}}\frac{g'_{An}}{g'_{Am}}\right]} = \frac{\alpha}{\left[1+\frac{g_{Am}}{g_{An}}\frac{g'_{An}}{g'_{Am}}\right]}\frac{1}{g_y^{\sigma}\beta^{-1}-1+\delta_k} \tag{A.59}$$

Using the law of motion for capital we can write

$$\frac{X}{Y} = \frac{K}{Y}(g_y - 1 + \delta_k) \tag{A.60}$$

thus we obtain

$$\tilde{x} = \frac{\alpha}{\left[1 + \frac{g_{Am}}{g_{An}} \frac{g'_{An}}{g'_{Am}}\right]} \frac{g_y - 1 + \delta_k}{g_y^{\sigma} \beta^{-1} - 1 + \delta_k} \tag{A.61}$$

- Resource Costs: following the same track as above, we rewrite Y as

$$Y_t = \left[(1 - \delta_y) \frac{g_{An}}{g_{Am}} \frac{g'_{Am}}{g'_{An}} + (1 - \delta_y)\right]^{\frac{\epsilon}{\epsilon - 1}} A_{mt} \tilde{M}_t \tag{A.62}$$

Plugging this into the expression for MPM yields

$$\left[(1 - \delta_y) \frac{g_{An}}{g_{Am}} \frac{g'_{Am}}{g'_{An}} + (1 - \delta_y)\right]^{\frac{1}{\epsilon - 1}} (1 - \delta_y) A_{mt} = (1 + \tau_v) A_{vt} \bar{V}_{t-1}^{\psi} \tag{A.63}$$

$$Y_t^{\frac{1}{\epsilon}} (1 - \delta_y)(A_{mt} \tilde{M}_t)^{\frac{\epsilon - 1}{\epsilon}} (1 - \delta_R) \left(\frac{V_t}{\tilde{M}_t}\right)^{\frac{\epsilon_m - 1}{\epsilon_m}} = V_t (1 + \tau_v) A_{vt} \bar{V}_t^{\psi} \tag{A.64}$$

Multiplying both sides by $V_t = \tilde{v} M_t$ we obtain

$$\left[(1 - \delta_y) \frac{g_{An}}{g_{Am}} \frac{g'_{Am}}{g'_{An}} + (1 - \delta_y)\right]^{\frac{1}{\epsilon - 1}} (1 - \delta_y) A_{mt} \tilde{v} M_t = V_t (1 + \tau_v) A_{vt} \bar{V}_{t-1}^{\psi} \tag{A.65}$$

Solving for $A_{mt} M_t$ and plugging into the Y_t expression, then rearranging

$$\frac{V_t A_{vt} \bar{V}_{t-1}^{\psi}}{Y_t} = \frac{\tilde{v}}{(1 + \tau_v) \left[1 + \frac{g_{An}}{g_{Am}} \frac{g'_{Am}}{g'_{An}}\right]} \tag{A.66}$$

We can follow the same tack for the cost of recovering waste. Start with the FOC for S_{ut}:

$$\left[(1 - \delta_y) \frac{g_{An}}{g_{Am}} \frac{g'_{Am}}{g'_{An}} + (1 - \delta_y)\right]^{\frac{1}{\epsilon - 1}} (1 - \delta_y) A_{mt} \delta_m R_t \alpha_r = \hat{c} S_{ut}$$

$$\left[(1 - \delta_y) \frac{g_{An}}{g_{Am}} \frac{g'_{Am}}{g'_{An}} + (1 - \delta_y)\right]^{\frac{1}{\epsilon - 1}} (1 - \delta_y) A_{mt} \delta_m (1 - \tilde{v}) M_t \alpha_r = \hat{c} S_{ut}$$

Rearranging to solve for $A_{mt}M_t$ we obtain

$$Y_t = \left[(1-\delta_y)\frac{g_{An}}{g_{Am}}\frac{g'_{Am}}{g'_{An}} + (1-\delta_y)\right]\frac{\hat{c}}{(1-\tilde{v})c\alpha_r(1-\delta_y)}cS_{ut} \tag{A.67}$$

Rearranging to solve for cost in terms of output:

$$\frac{cS_{ut}}{Y_t} = \left(\frac{(1-\tilde{v})c\alpha_r}{\hat{c}}\right)\frac{1}{\left[\frac{g_{An}}{g_{Am}}\frac{g'_{Am}}{g'_{An}} + 1\right]} \tag{A.68}$$

Thus we can write

$$\begin{aligned}
\tilde{q} &= 1 - \tilde{x} - \frac{V_t A_{vt}\bar{V}_{t-1}^{\psi}}{Y_t} - \frac{cS_{ut}}{Y_t} \\
\tilde{q} &= 1 - \frac{\alpha}{\left[1 + \frac{g_{Am}}{g_{An}}\frac{g'_{An}}{g'_{Am}}\right]}\frac{g_y - 1 + \delta_k}{g_y^{\sigma}\beta^{-1} - 1 + \delta_k} - \frac{\tilde{v}}{(1+\tau_v)\left[1 + \frac{g_{An}}{g_{Am}}\frac{g'_{Am}}{g'_{An}}\right]} \\
&\quad - \left(\frac{(1-\tilde{v})c\alpha_r}{\hat{c}}\right)\frac{1}{\left[1 + \frac{g_{An}}{g_{Am}}\frac{g'_{Am}}{g'_{An}}\right]}
\end{aligned}$$

This expression makes clear the role of policy. When subsidies rise, the last term in the expression grows, lowering $\tilde{q}$, and thus raising the recycling rate as seen in Equation A.54. By contrast, when τ_v rises, the second to last term becomes smaller, raising $\tilde{q}$ and lowering the recycling rate. This is part 2 of the proposition.

■

A.3 Calibrated Parameters

A.3.1 $\delta_C, \delta_K, C_0, K_0$: Depreciation rates and initial stocks

Most of the data required to implement our framework is directly available, however our treatment of the material stock of consumption as durable implies that the variable C_t is a theoretical construct and, as such, is not directly measured in NIPA. Measured consumption in NIPA is not C_t, but rather Q_t (additions to the stock of consumption).[26] We assume a homogeneous consumption good Q_t and a closed economy without a foreign sector. Hence, the economy's current period output is divided between consumption goods (Q_t) and investment goods (X_t). That is, $Y_t = Q_t + X_t$, where Y_t is defined as the economy's gross

[26]The aggregate materials share of the consumption stock $\bar{\gamma}_t$ follows its own *materials law of motion* (MLOM) described in Equation (3). Notice that at the extreme case of entirely non-durable consumption with $\delta_c = 1$, equation (2) implies $C_t = Q_t$ and then equation (3) yields $\bar{\gamma}_t = \gamma_t$.

domestic product (GDP). We follow Kehoe and Prescott (2007) in allocating net exports to consumption and distributing government expenditures to consumption and investment goods.[27] Further, because our homogeneous consumption good represents goods and services with varying depreciation rates, we treat materials in consumption as durable to allow our representation of the "average" consumption good in the economy to have a depreciation rate less than one. The stock of consumption goods depreciates at rate δ_C and follows the law of motion as in Equation (2). The capital stock follows a similar law of motion but we ignore it here since we will treat consumption as the only source of waste.

In order to construct the path of consumption stock we therefore require an initial condition C_0 and depreciation rate δ_C. To compute these values we follow standard calibration procedures. Under the assumption that the economy is on a balanced growth path, the consumption output ratio C/Y should be constant, as should the share of depreciated consumption $\delta_C C/Y$. These two conditions yields two equations that can be jointly solved for the unknowns C_0 and δ_C.

We define year 0 to be 1960. To pin down C_0 we require that the consumption-output ratio of the initial period, i.e., C_0/Y_0, to match the average consumption-output ratio over our reference period 1960 - 1970. Hence, we choose the consumption stock so that the consumption-output ratio in 1960 matches its average over 1961-1970:

$$\frac{C_{1960}}{Y_{1960}} = \frac{1}{10} \sum_{t=1961}^{1970} \frac{C_t}{Y_t} \tag{A.69}$$

Second, we choose δ_C to be consistent with the average ratio of depreciation to GDP observed in the data over the period used for calibration purposes. We use data for the period 1960-1970 on Durable Consumption Depreciation (DCD_t) and Non-Durable (real) Consumption (NDC_t) to compute the path of consumption depreciation (CD_t), as follows: $CD_t = DCD_t + NDC_t$. That is, the depreciated consumption in a given year is the sum of durable consumption depreciation as measured by the U.S. Bureau of Economic Analysis (BEA) plus all non-durable consumption in the same year. The underlying assumption here is that non-durable goods fully depreciate in a given year (i.e., a depreciation rate equals to one). Also, because NIPA only provides a breakdown between durable and non-durable for Personal Consumption Expenditures (PCE), we apply the shares of durable and non-durable from PCE to both government consumption (GC) and net export (NX). We find that the average share of durable PCE during this period is equal to 0.15. Hence, our NDC_t measure is $NDC_t = (1 - 0.85) \times (PCE_t + GC_t + NX_t)$.

[27]Notice that although we have a government in our economy, the role of the government is to subsidize the recycler's collection cost.

For the United States, we find that the ratio of consumption depreciation to GDP over the period 1970 - 2015 is

$$\frac{1}{10}\sum_{t=1970}^{2015}\frac{\delta_C C_t}{Y_t} = \frac{1}{10}\sum_{t=1970}^{2015}\frac{CD_t}{Y_t} = 0.6838 \tag{A.70}$$

Notice that because equations (A.69)-(A.70) depend on the path of the consumption stock C_t, they both depend on C_0, δ_C together and must be solved simultaneously. Then, for any given C_0 and δ_C, the consumption law of motion, equation (2) and the path of observed consumption investment (Q_t) allow us to construct a path of consumption stock C_t for the period of interest. More precisely, the system of equations (2), (A.70), and (A.69) allow us to use data on consumption investment, Q_t, to solve for the sequence of consumption stocks and for the depreciation rate δ_C. Solving this system of equations, we obtain the sequence of consumption stocks and a calibrated value for consumption depreciation, $\delta_C = 0.155$.

A.3.2 ρ: Share of materials embodied in units of output

The parameter $\rho_t \in (0,1)$ allows for the possibility that only a certain fraction of materials extracted remains in the economy after production and consumption. Since materials used in production can come from abroad or be extracted domestically, we use the MFA measure Domestic Material Consumption (DMC) to capture the mass of material inflows to the U.S. economy. DMC is the total materials consumed in the domestic economy, computed as the sum of domestic extraction (DE) and net exports.[28] In general ρ_t will not be unity, in part because of measurement differences. MFA counts inputs at extraction and so records a large amount of material by weight that does not end up in final consumption.[29] To account for these, and many other, deviations of materials from the production-to-MSW stream we assume that only a share ρ_t of materials are passed from production to consumption, and then to waste, while a share $1-\rho_t$ flows into other, unmeasured, waste streams. We set $\rho = 0.254$, which is the average for the time period studied (Figure 3, right panel).

A.3.3 ψ: Elasticity virgin materials extraction

The virgin materials producer profit maximization implies

$$p_{v,t} = A_{v,t}\bar{V}_t^{\psi}.$$

[28] In this paper we have assumed a single final goods producer to keep the framework as close as possible to a standard growth model, but there is no reason ex-ante to assume consumption and investment goods feature the same material requirements.

[29] For example, iron extraction is measured at "run-of-the-mill", i.e. the amount of iron ore initially extracted from a mine. However, the conversion rate of that gross ore to ore concentrate is approximately 82-to-1 approximately (Sept 2013 Eurostat EWMFA compilation guide), so that less than 1.5% of that extracted weight will actually be used in production of goods and services.

The parameter $\psi > 0$ gives the elasticity of virgin materials extraction costs with respect to cumulative extraction and extraction costs fall with the productivity term $A_{v,t}$.

Taking logs of both sides of $p_{v,t} = A_{v,t}\bar{V}_t^{\psi}$, we obtain

$$ln(p_{v,t}) = ln(A_{v,t}) + \psi ln(\bar{V}_t)$$
$$ln(p_{v,t-1}) = ln(A_{v,t-1}) + \psi ln(\bar{V}_{t-1})$$

Subtracting one equation from the other, we can write

$$\begin{aligned} ln\left(\frac{p_{v,t}}{p_{v,t-1}}\right) &= ln\left(\frac{A_{v,t}}{A_{v,t-1}}\right) + \psi ln\left(\frac{\bar{V}_t}{\bar{V}_{t-1}}\right) \\ ln\left(1 + \frac{p_{v,t} - p_{v,t-1}}{p_{v,t-1}}\right) &= ln\left(1 + \frac{A_{v,t} - A_{v,t-1}}{A_{v,t-1}}\right) + \psi ln\left(1 + \frac{\bar{V}_t - \bar{V}_{t-1}}{\bar{V}_{t-1}}\right) \\ \underbrace{\ln\left(1 + g_{p_{v,t}}\right)}_{y} &= \underbrace{\ln\left(1 + g_{A_{v,t}}\right)}_{a} + b\underbrace{\ln\left(1 + g_{\bar{V}_t}\right)}_{x} \end{aligned} \tag{A.71}$$

We estimate a and b from equation (A.71), using data on the historical prices of virgin materials $p_{v,t}$ and a constructed measure for $\bar{V}_t$, which is the sum of all materials extracted up to period t. Our values of $\bar{V}_t$ are based on the global material extraction data (DE) by material group and by use type, available from Krausmann et al. (2018). We consider data for biomass, ores, and non-metallic minerals, consistent with our measure of virgin materials prices discussed in Section A.1.1. Starting in 1901, we calculate $\bar{V}_t = \sum_{j=0}^{t-1} V_j$. We then use the produced series for the period 1980-2015. Quantities are converted from Gigton/year to tons/year. Hence, the calculated value of a implies that the gross growth rate of virgin materials TFP is 0.969, i.e., $g_{A_{v,t}} = -0.0269$. Also, we interpret $b = \psi$ and set our benchmark value $\psi = 1.1178$.[30]

A.3.4 δ_R: Recycled materials share

In order to calibrate δ_R, the recycled materials share in the composite material aggregate

$$\tilde{M}_t = \left[\delta_R R_t^{\frac{\epsilon_m - 1}{\epsilon_m}} + (1-\delta_R) V_t^{\frac{\epsilon_m - 1}{\epsilon_m}}\right]^{\frac{\epsilon_m}{\epsilon_m - 1}},$$

we compute $(1-\delta_R)$ as average expenditure share for virgin materials as follows:

$$(1-\delta_R) = \frac{P_{vt}V_t}{(P_{vt}V_t + P_{rt}R_t)}.$$

[30]We consider growth rates for different time intervals, e.g., current period to immediately previous period ($t = 1$), current period to second previous period ($t = 2$), and so on up to $t = 10$, i.e., $(p_{v,t} - p_{v,t-10})/p_{v,t-10}$. Our reported values are based on $t = 8$ and $t = 9$, due to the robustness of the calculated values a and b.

This implies that $\delta_R = 0.0065$, our benchmark value.

A.3.5 α_R: Marginal returns of the recover process

We begin by discussing how $\hat{c}_t = \bar{\gamma}_{t-1}(c_t - \tau_t)$ is constructed. According to Bohm et al. (2010) (Table 1), we see that the average quantity of material collected for recycling in 1996 is 3232 tonnes and average cost is 371,060, yielding an average cost per tonne of \$114.81 in 1996 dollars. Using the GDP deflator, this is equivalent to an average cost per tonne of \$173.31 (2018 dollars), which we assume is constant for the period $t = 1971 - 2015$. Next, we calculate T_t^d, the total expenditure on municipal recycling programs. Also according to Bohm et al. (2010) the average expenditure on waste is \$2,290,700 and average expenditure on recycling is \$371,060. Jointly, this suggests that 13.94% of MSW expenditure locally is given to recycling in 1996. If we assume this share is constant, then we can compute the total expenditure in recycling from municipalities and states as $T_t^d = (\%MSWexp \times Y_t) \times 0.1394$. In this exercise we are assuming that (i) private and public recycling firms have access to the same recycling technology and (ii) any public expenditure on recycling is freely given to the private sector (because we don't see any revenue coming in, only expenditures on waste management, we treat this as effectively a transfer; an upper bound subsidy to recycling firms). Then, spreading the total expenditure on municipal recycling programs (T_t^d) over the total tons of waste collected (W_t), i.e., $\bar{\gamma}_{t-1}W_t$, we obtain $\tau_t = \frac{T_t^d}{\bar{\gamma}_{t-1}W_t}$. Hence, for each period t, we obtain the net cost per ton as the difference between the fix cost c_t and τ_t. The final step is to convert the net cost per ton into the net cost per unit of depreciated consumption S_{ut}. If each consumption unit has $\bar{\gamma}_{t-1}$ tons and each ton costs $c_t - \tau_t$ (times $100,000$ to obtain the net cost per $100,000$ tons to make it comparable with our waste measure), then the per unit cost is $\hat{c}_t = \bar{\gamma}_{t-1}(c_t - \tau_t)$.

We now turn to estimating α_r. The solution of the recycler's problem implies the following representation of recycled materials:

$$R_t = A_{rt}^{\frac{1}{1-\alpha_r}} \left(\frac{\bar{\gamma}_t p_{rt} \alpha_r}{\hat{c}_t} \right)^{\frac{\alpha_r}{1-\alpha_r}}$$

Taking logs of the growth rate of recycled materials (R_{t+1}/R_t) we obtain

$$\underbrace{\ln\left(\frac{R_{t+1}}{R_t}\right)}_{y} = \underbrace{\left(\frac{1}{1-\alpha_r}\right)\ln\left(\frac{A_{rt+1}}{A_{rt}}\right)}_{a} + \underbrace{\left(\frac{\alpha_r}{1-\alpha_r}\right)}_{b}\underbrace{\ln\left(\frac{p_{rt+1}}{p_{rt}}\frac{\hat{c}_t}{\hat{c}_{t+1}}\frac{\bar{\gamma}_{t+1}}{\bar{\gamma}_t}\right)}_{x} \tag{A.72}$$

The intuition for the exercise is that the A_r terms should move relatively slowly on an annual basis, so that in log terms this should be close to zero. Estimating this equation, we obtain

$a = 0.049$ and $b = 0.445$. This implies a benchmark value for α_r equal to 0.3053, since we interpret $b = \alpha_r/(1-\alpha_r)$.

A.3.6 ε_m: Elasticity of substitution materials composite

Consider the first-order conditions of the final goods producer problem - equations (A.11) and (A.12), assuming $\tau_v = 0$ - presented here again for convenience:

$$p_{v,t} = Y_t^{\frac{1}{\epsilon}}(1-\delta_y)(A_{mt}\tilde{M}_t)^{\frac{\epsilon-1}{\epsilon}}(1-\delta_R)\left(\frac{V_t}{\tilde{M}_t}\right)^{\frac{\epsilon_m-1}{\epsilon_m}}$$

$$p_{r,t} = Y_t^{\frac{1}{\epsilon}}(1-\delta_y)(A_{mt}\tilde{M}_t)^{\frac{\epsilon-1}{\epsilon}}(\delta_R)\left(\frac{R_t}{\tilde{M}_t}\right)^{\frac{\epsilon_m-1}{\epsilon_m}}$$

Combining these two equations and taking logs, we have

$$\underbrace{\ln\left(\frac{V_t}{R_t}\right)}_{y} = \underbrace{\left(\frac{\epsilon_m}{\epsilon_m - 1}\right)}_{a}\underbrace{\ln\left(\frac{p_{v,t}}{p_{r,t}}\frac{\delta_R}{(1-\delta_R)}\right)}_{x}$$

Estimating a simple OLS regression, we obtain $a = -1.0522$ (statistically significant at 1%), which implies $\epsilon_m = 0.5127$.

A.3.7 δ_Y: Materials expenditure share

Final goods (Y_t) are produced by combining capital and labor with a composite material aggregate $\tilde{M}_t$ according to the following production function

$$Y_t = \left(\delta_Y(A_{n,t}K_t^{\alpha}L_t^{1-\alpha})^{\frac{\epsilon-1}{\epsilon}} + (1-\delta_Y)(A_{m,t}\tilde{M}_t)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}},$$

We compute $(1-\delta_Y)$ as average expenditure share for materials

$$(1-\delta_Y) = \frac{P_{vt}V_t + P_{rt}R_t}{Y_t}.$$

This implies that $\delta_Y = 0.8767$, our benchmark value.

A.3.8 ε: Elasticity of substitution capital/labor and materials

We follow Hassler et al. (2012) in order to estimate the elasticity of substitution between capital/labor and materials (ε). We assume that final goods are produced by combining capital and labor with a composite material aggregate $\tilde{M}$ according to

$$Y_t = \left(\delta_y(A_{n,t}K_t^{\alpha}L_t^{1-\alpha})^{\frac{\epsilon-1}{\epsilon}} + (1-\delta_y)(A_{m,t}\tilde{M}_t)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}},$$

Under perfect competition in input markets, marginal products equal factor prices, so that labor's and materials' shares of income are respectively given by

$$L_t^{share} \equiv \frac{\partial Y_t}{\partial Ł_t}\frac{L_t}{Y_t} = (1-\alpha)\delta_y \left(\frac{A_{n,t}K_t^{\alpha}L_t^{1-\alpha}}{Y_t}\right)^{\frac{\epsilon-1}{\epsilon}} \tag{A.73}$$

$$\tilde{M}_t^{share} \equiv \frac{\partial Y_t}{\partial \tilde{M}_t}\frac{\tilde{M}_t}{Y_t} = (1-\delta_y)\left(\frac{A_{m,t}\tilde{M}_t}{Y_t}\right)^{\frac{\epsilon-1}{\epsilon}} \tag{A.74}$$

Equations (A.73) and (A.74) can be rearranged and solved directly for the two technology trends $A_{n,t}$ and $A_{m,t}$. This delivers

$$A_{n,t} = \frac{Y_t}{K_t^{\alpha}L_t^{1-\alpha}}\left(\frac{L_t^{share}}{(1-\alpha)\delta_y}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{A.75}$$

$$A_{m,t} = \frac{Y_t}{\tilde{M}_t}\left(\frac{\tilde{M}_t^{share}}{1-\delta_y}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{A.76}$$

Note that with ε and δ_y given, and with data on Y_t, K_t, L_t, $\tilde{M}_t$, L_t^{share} and $\tilde{M}_t^{share}$, equations (A.73) and (A.74) give explicit expressions for the evolution of the two technologies.

We now estimate the elasticity ε, together with some other parameters, directly with a maximum-likelihood approach. We specify that the technology series are exogenous processes of a certain form and then estimate the associated parameters along with ε. The technology processes have innovation terms and the maximum likelihood procedure, of course, chooses these to be small. Hence, the key assumption behind the estimation is to find a value of ε such that the implied technology series behave smoothly, or as smoothly as the data allows.

$$\begin{bmatrix} a_t \\ a_t^{\tilde{M}} \end{bmatrix} - \begin{bmatrix} a_{t-1} \\ a_{t-1}^{\tilde{M}} \end{bmatrix} = \begin{bmatrix} \theta^A \\ \theta^{\tilde{M}} \end{bmatrix} + \begin{bmatrix} \omega_t^A \\ \omega_t^{\tilde{M}} \end{bmatrix} \tag{A.77}$$

where $a_t = \log(A_{n,t})$ and $a_m^{\tilde{M}} = \log(A_{m,t})$, and $\boldsymbol{\omega_t} \equiv \begin{bmatrix} \omega_t^A \\ \omega_t^{\tilde{M}} \end{bmatrix} \sim \mathcal{N}(0,\Sigma)$.

Dividing equations (A.75) and (A.76) by their counterparts in period $t-1$ gives

$$\frac{A_{n,t}}{A_{n,t-1}} = \frac{Y_t}{K_t^{\alpha}L_t^{1-\alpha}}\frac{K_{t-1}^{\alpha}L_{t-1}^{1-\alpha}}{Y_{t-1}}\left(\frac{L_t^{share}}{L_{t-1}^{share}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{A.78}$$

$$\frac{A_{m,t}}{A_{m,t-1}} = \frac{Y_t}{\tilde{M}_t}\frac{\tilde{M}_{t-1}}{Y_{t-1}}\left(\frac{\tilde{M}_t^{share}}{\tilde{M}_{t-1}^{share}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{A.79}$$

Taking logs of (A.78) and (A.79) and using (A.77) in these expressions gives allows us to write the system as

$$s_t = \boldsymbol{\theta} - \frac{\epsilon}{\epsilon - 1} z_t + \boldsymbol{\omega_t} \tag{A.80}$$

where $s_t \equiv \begin{bmatrix} \log\left(\frac{Y_t}{K_t^{\alpha} L_t^{1-\alpha}}\right) - \log\left(\frac{Y_{t-1}}{K_{t-1}^{\alpha} L_{t-1}^{1-\alpha}}\right) \\ \log\left(\frac{Y_t}{\tilde{M}_t}\right) - \log\left(\frac{Y_{t-1}}{\tilde{M}_{t-1}}\right) \end{bmatrix}$ and $z_t \equiv \begin{bmatrix} \log\left(L_t^{share}\right) - \log\left(L_{t-1}^{share}\right) \\ \log\left(\tilde{M}_t^{share}\right) - \log\left(\tilde{M}_{t-1}^{share}\right) \end{bmatrix}$.

The log-likelihood function is given by

$$\begin{aligned} l(s|\boldsymbol{\theta}, \varepsilon, \Sigma) = {} & constant - \frac{N}{2} \log |\Sigma| \\ & - \frac{1}{2} \sum_{t=1}^{N} \left(s_t - \left(\boldsymbol{\theta} - \frac{\epsilon}{\epsilon - 1} z_t \right) \right)^T \Sigma^{-1} \left(s_t - \left(\boldsymbol{\theta} - \frac{\epsilon}{\epsilon - 1} z_t \right) \right) \end{aligned} \tag{A.81}$$

Maximization of equation (A.81) with respect to $\boldsymbol{\theta}$, ε, and Σ gives the estimated parameters. The data for output, virgin and recycled materials, and their prices was discussed above. Data on the labor force is taken from the Bureau of Labor Statistics, whereas data on the capital stock as well as the data required to compute labor's share of income are taken from the Bureau of Economic Analysis. The results of our estimation are displayed in Table A.1.

Table A.1: Estimated Parameters

θ^{A}	$\theta^{\tilde{M}}$	ε
0.49460	0.22808	0.2555
(0.00796)	(0.01014)	(0.01237)

Note: Standard errors in parentheses.

A.3.9 TFP and TFP Growth Rates

The solution to the virgin primary materials producers profit maximization problem implies that its TFP can be written as

$$A_{v,t} = \frac{p_{v,t}}{\bar{V}_t^{\psi}} \tag{A.82}$$

In order to calculate $A_{r,t}$, first we need to calculate the amount of depreciated consumption units the recycler chooses to recover (S_{ut}). The single firm profit maximization problem

(Section A.2.7) first-order condition with respect to S_{ut} implies

$$S_{ut} = \frac{Y_t^{\frac{1}{\epsilon}}(1-\delta_y)(A_{mt}\tilde{M}_t)^{\frac{\epsilon-1}{\epsilon}}\delta_R\alpha_r\left(\frac{R_t}{\tilde{M}_t}\right)^{\frac{\epsilon_m-1}{\epsilon_m}}}{\hat{c}}$$

Given a series S_{ut}, we calculate the recycled primary materials producer TFP, consistent with equation (11 - $R_t = A_{r,t}\left(W_t s_t\right)^{\alpha_r}$ - as follows

$$A_{rt} = \frac{R_t}{(\bar{\gamma}_{t-1}S_{ut})^{\alpha_r}}$$

Next, the single firm profit maximization problem (Section A.2.7) first-order condition with respect to V_t implies that the materials composite is as follows:

$$A_{mt} = \left(\frac{V_t A_{vt}\bar{V}_t^{\psi}}{Y_t^{\frac{1}{\epsilon}}(1-\delta_y)(1-\delta_R)\left(\frac{V_t}{\tilde{M}_t}\right)^{\frac{\epsilon_m-1}{\epsilon_m}}}\right)^{\frac{\epsilon}{\epsilon-1}}\frac{1}{\tilde{M}_t}$$

And finally, we use the final good production function to calculate the capital-labor composite TPF, defined as follows:

$$A_{nt} = \left(\frac{(Y_t)^{\frac{\varepsilon-1}{\varepsilon}} - (1-\delta_Y)(A_{m,t}\tilde{M}_t)^{\frac{\epsilon-1}{\epsilon}}}{\delta_Y(K_t^{\alpha}L_t^{1-\alpha})^{\frac{\epsilon-1}{\epsilon}}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

The calculated series of TFP - $A_{vt}, A_{rt}, A_{mt}, A_{nt}$ - for the period 1980-2015 are presented in Figure A.7. And, the growth rates of virgin materials, recycled materials, materials composite and capital-labor composite, i.e., $g_{A_v}, g_{A_r}, g_{A_m}, g_{A_n}$, respectively are presented in Table A.2.

Table A.2: TFP Growth Rates

g_{Av}	g_{Ar}	g_{Am}	g_{An}
0.969	1.0561	1.0317	1.0056

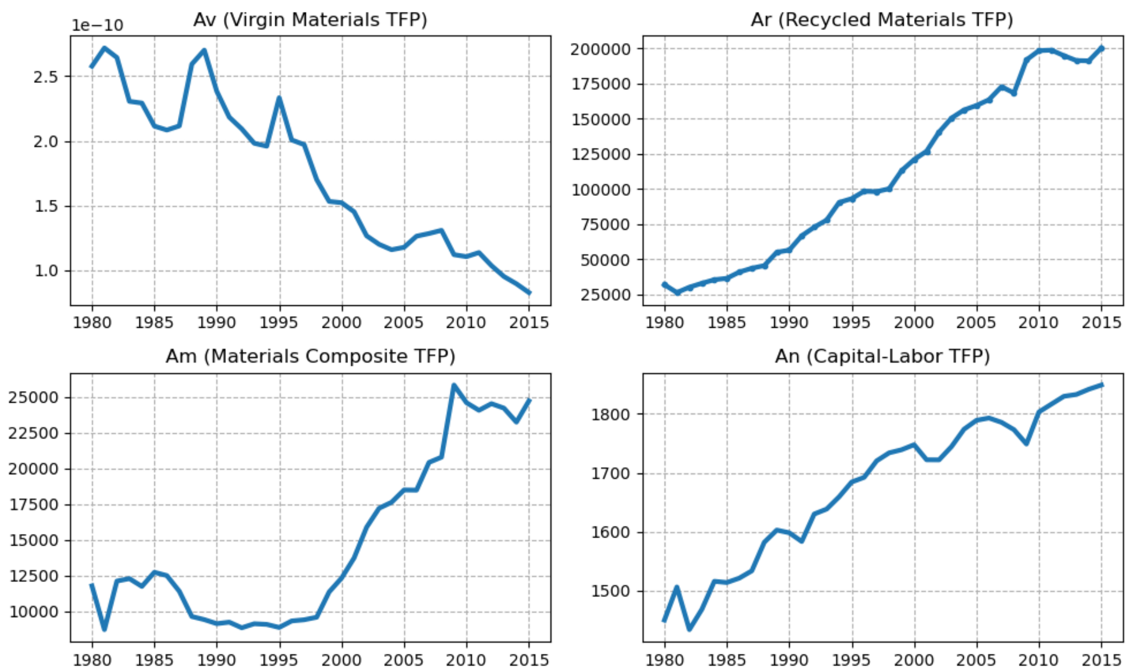

Figure A.7: $A_{vt}, A_{rt}, A_{mt}, A_{nt}$ TFP Series

A.3.10 $\eta_N, \eta_{\tilde{M}}, \eta_R, \lambda$: Calibrated DTC Parameters

Table A.3: Intensive Form
Exogenous Virgin Material Prices

	Variable	1980	2015
TFP			
Recycled Materials	A_r	32046.10	200360.15
Material Composite	A_m	11787.10	24701.70
Final Goods	A_n	1449.85	1848.72
Labor (BLS Employment)	L	90532000	141804000
Intensive Form			
Recycled Materials TFP	$\hat{A}_r$	0.00625	0.01734
Material Composite TFP	$\hat{A}_m$	8.11423	24.06670
Output	Y	1.12837	1.27443
Stock of Consumption	C	3.91173	5.37349
Consumption Flow	Q	0.70537	0.92749
Capital Stock	K	1.49741	1.87154
Investment	X	0.22876	0.24573
Processed Recycled Materials	S_u	2.64925	0.82657
Materials so far Extracted	$\bar{V}$	56.12102	35.29941
Virgin Materials	V	0.15621	0.07273
Recycled Materials	R	0.00319	0.00461
Materials Composite	$\tilde{M}$	0.12350	0.06700
Average Material Share	$\bar{\gamma}$	0.04127	0.01581
Material Share	γ	0.03987	0.01626

Note: Python file: ArbexMahone_CalibrationMaster.

Table A.4: Intensive Form
Endogenous Virgin Material Prices

	Variable	1980	2015
TFP			
Virgin Materials	A_v	2.57766e-10	8.26965e-11
Recycled Materials	A_r	32046.1	200360.15
Material Composite	A_m	11787.1	24701.7
Final Goods	A_n	1449.85	1848.72
Labor (BLS Employment)	L	90532000	141804000
Intensive Form			
Recycled Materials TFP	$\hat{A}_r$	0.00142847	0.00469623
Material Composite TFP	$\hat{A}_m$	67.9908	157.885
Output	Y	1.12837	1.27443
Stock of Consumption	C	3.91173	5.37349
Consumption Flow	Q	0.70537	0.92749
Capital Stock	K	1.49741	1.87154
Investment	X	0.22876	0.24573
Processed Recycled Materials	S_u	2.64925	0.82657
Materials so far Extracted	$\bar{V}$	6.6977	5.3808
Virgin Materials	V	0.0186	0.0111
Recycled Materials	R	0.0004	0.0007
Materials Composite	$\tilde{M}$	0.0147	0.0102
Average Material Share	$\bar{\gamma}$	0.0049	0.0024
Material Share	γ	0.0048	0.0025

Note: Python file: ArbexMahone_CalibrationMaster.

A.4 Dynamics and Circularity in the Short Run

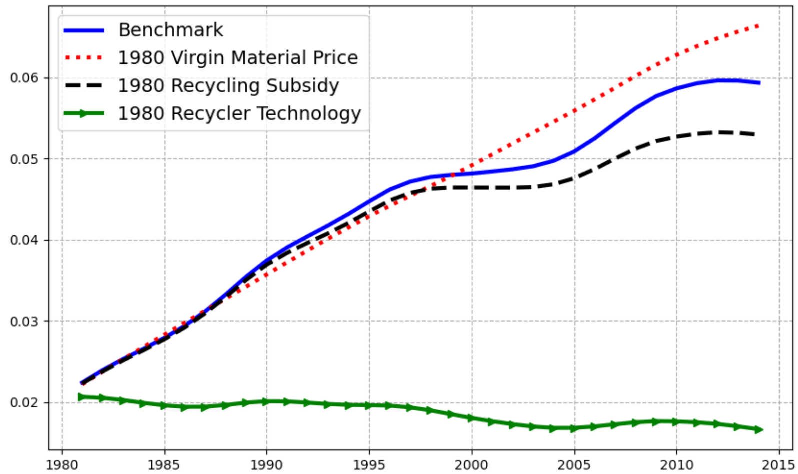

Figure A.8: Short Run Material Mix Dynamics (De-trended Series), 1980-2100.

A.5 Achieving an Exogenous Recycling Rate Target

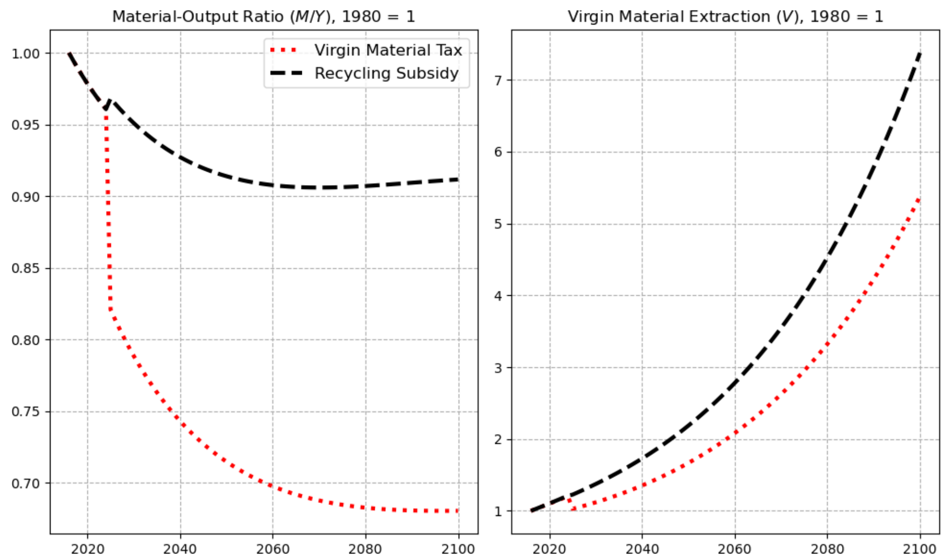

Figure A.9: Material-Output Ratio and Virgin Materail Extraction - Virgin Material Tax and Recycling Subsidy - Policy Relative to Benchmark, 1980-2100, 1980 = 1.

A.6 Robustness - Recycling Subsidy

Table A.5: Robustness - Recycling Subsidy

Description		Parameter	Value	Recycling Subsidy (%)
Benchmark[1]				86.96
Research decreasing returns parameter[2]		λ	0.40	88.80
		λ	0.50	83.10
Research Efficiency				
	Capital-Labor	η_n	0.0063	85.95
		η_n	0.0077	87.17
	Materials Composite	η_m	0.0405	86.36
		η_m	0.0495	86.76
	Recycled Materials	η_r	0.1845	87.99
		η_r	0.2255	85.13

Notes: (1) Benchmark parameters: $\lambda = 0.45$, $\eta_n = 0.007$, $\eta_m = 0.045$, $\eta_r = 0.205$ (Table I). $\lambda = 0.50$ (Acemoglu et al., 2018; Akcigit and Kerr, 2018); Casey (2023) estimates $\lambda = 0.40$. Benchmark case requires a recycling subsidy covering 86.96% of costs in the endogenous price model to obtain a 50% recycling rate by 2030.